〈1-1〉 (1.2.1)
[¥mbox {v}]=¥mbox{L}¥mbox{T}^{-1}

〈1-2〉 (1.3.1)
y=ax+b

〈1-3〉 (1.3.2)
y=b¥sin(ax)+c

〈1-4〉 (1.3.3)
y=f(x)

〈1-5〉 (1.3.4)
y=f(x_1,x_2,x_3,¥cdots)

〈1-6〉 (1.3.5)
y=x^2

〈1-7〉
4=x^2

〈1-8〉 (1.3.6)
y^2=x

〈1-9〉 (1.3.7)
x=f^{-1}(y)

〈1-10〉 (1.3.8)
y=f(x)=ax+b

〈1-11〉 (1.3.9)
x=f^{-1}(y)=¥frac{y-b}{a}

〈1-12〉 (1.3.10)
```
¥begin{array}{l}
y=¥left¥{
¥begin{array}{l}
e^x ¥¥ ¥¥ ¥sin x ¥¥ ¥¥ ¥sin(x^3) ¥¥ ¥¥
(¥sin x)^{1/3}
¥end{array} ¥right.
```

<1-13> (1.3.11)

```
¥begin{array}{l}
y=¥left¥{
¥begin{array}{l}
¥ln y ¥¥ ¥¥ ¥sin^{-1}y ¥¥ ¥¥
¥sqrt[3]{¥sin^{-1}y} ¥¥ ¥¥
¥sin^{-1}(y^3)
¥end{array}¥right.
```

<1-13> (1.3.11)

```
¥begin{array}{rl}
x&=f^{-1}(y) ¥¥ ¥¥
&=¥left¥{
¥begin{array}{l}
¥ln y ¥¥ ¥¥
¥sin^{-1}y ¥¥ ¥¥
¥sqrt[3]{¥sin^{-1}y} ¥¥ ¥¥
¥sin^{-1}(y^3)
¥end{array} ¥right. ¥¥ ¥¥
¥end{array}
¥end{array}
```

<2-1> (2.1.1)

```
¥lim_{¥Delta¥rightarrow0}
¥left[¥frac{¥Delta f(x)}{¥Delta x}¥right]
=¥frac{df(x)}{dx}¥equiv f^¥prime(x)
```

<2-2> (2.1.2)

```
¥begin{array}{l}
¥displaystyle{¥Delta f(x)=¥frac{df(x)}{dx}¥Delta x}
¥¥ ¥¥
¥displaystyle{¥mbox{or}¥quad df(x)
=¥frac{df(x)}{dx}dx}
¥end{array}
```

<2-3>

```
(f¥pm g)^¥prime=f^¥prime¥pm g^¥prime
```

<2-4>

(kf)^¥prime=kf^¥prime

〈2-5〉
(fg)^¥prime=f^¥prime g+fg^¥prime

〈2-6〉
¥frac{dy}{dx}=¥frac{dy}{dz}¥frac{dz}{dx}=
¥frac{df(z)}{dz}¥frac{dg(x)}{dx}

〈2-7〉 (2.1.3)
¥frac{d^nf(x)}{dx^n}¥equiv f^{(n)}(x),
¥quad(n=0,1,2,¥cdots)

〈2-8〉 (2.1.4)
¥frac{df(x)}{dx}=¥frac{d}{dx}f(x)

〈2-9〉 (2.1.5)
¥begin{array}{rl}
¥displaystyle{¥frac{df(x)}{dx}}
&=¥displaystyle{¥lim_{¥Delta x¥rightarrow0}
¥left[¥frac{f(x+¥Delta x)-f(x)}{¥Delta x}¥right]}}
¥¥ ¥¥
&=¥displaystyle{¥lim_{¥Delta x¥rightarrow0}
¥displaystyle{
¥left[¥frac{¥Delta f(x)}{¥Delta x}¥right]
}
¥end{array}

〈2-10〉 (2.1.6)
¥frac{df(x)}{dx}¥quad¥Rightarrow¥quad
¥frac{d}{dx}f(x)¥quad¥mbox{or}¥quad Df(x)

〈2-11〉 (2.2.1)
¥begin{array}{rl}
f(x)=
&f(a) ¥¥ ¥¥
&+¥displaystyle{¥frac{f^{(1)}(a)}{1!}(x-a)} ¥¥ ¥¥
&+¥displaystyle{¥frac{f^{(2)}(a)}{2!}(x-a)^2} ¥¥ ¥¥
&+¥cdots ¥¥ ¥¥
&+¥displaystyle{¥frac{f^{(n)}(a)}{n!}(x-a)^n} ¥¥ ¥¥

```
&+¥cdots
¥end{array}

〈2-12〉
n!=1¥times2¥times3¥cdots¥times(n-1)¥times n

〈2-13〉 (2.2.2)
¥begin{array}{rl}
f(x)=&f(0) ¥¥ ¥¥
&+¥displaystyle{¥frac{f^{(1)}(0)}{1!}x} ¥¥ ¥¥
&+¥displaystyle{¥frac{f^{(2)}(0)}{2!}x^2}} ¥¥ ¥¥
&+¥cdots ¥¥ ¥¥
&+¥displaystyle{¥frac{f^{(n)}(0)}{n!}x^n} ¥¥ ¥¥
&+¥cdots
¥end{array}

〈2-14〉 (2.2.3)
f(x)=f(0)+ f^{(1)}(0)x

〈2-15〉 (2.2.4)
(1+x)^n=1+ nx

〈2-16〉
e^x

〈2-17〉
1+x+¥frac{x^2}{2}

〈2-18〉
¥sin x

〈2-19〉
x-¥frac{x^3}{6}+¥frac{x^5}{120}

〈2-20〉
¥cos x

〈2-21〉
1-¥frac{x^2}{2}+¥frac{x^4}{24}
```

〈2-22〉

```
¥ln(1+x)
```

〈2-23〉

```
x-¥frac{x^2}{2}+¥frac{x^3}{3}
```

〈2-24〉

```
(1+x)^¥alpha
```

〈2-25〉

```
1+¥alpha x+¥frac{¥alpha(¥alpha-1)}{2}x^2
```

〈2-26〉 (2.2.5)

```
¥frac{d^2¥theta}{dt^2}=-¥frac{g}{¥ell}¥sin¥theta
```

〈2-27〉 (2.2.6)

```
¥frac{d^2¥theta}{dt^2}=-¥frac{g}{¥ell}¥theta
```

〈2-28〉 (2.2.7)

```
¥theta=A¥sin(¥omega t),¥quad
¥left(¥omega=¥sqrt{g/¥ell}¥right)
```

〈2-29〉 (2.3.1)

```
¥lim_{x¥rightarrow a}¥frac{f(x)}{g(x)}=
¥frac{f^{¥prime¥prime}(a)}{ g^{¥prime¥prime}(a)}
```

〈2-30〉

```
¥left¥{¥begin{array}{rl}
f(x)
&=¥displaystyle{
¥left[¥frac{df(x)}{dx}¥right]_{x=a}(x-a)} ¥¥ ¥¥
&+¥displaystyle{¥frac{1}{2}
¥left[¥frac{d^2f(x)}{dx^2}¥right]_{x=a}(x-a)^2}¥¥ ¥¥
g(x)
&=¥displaystyle{
¥left[¥frac{dg(x)}{dx}¥right]_{x=a}(x-a)} ¥¥ ¥¥
&+¥displaystyle{¥frac{1}{2}
¥left[¥frac{d^2g(x)}{dx^2}¥right]_{x=a}(x-a)^2}
¥end{array}¥right.
```

〈2-31〉
¥left¥{¥begin{array}{rl}
f(x)
&=¥displaystyle{
¥left[¥frac{df(x)}{dx}¥right]_{x=a}(x-a)} ¥¥ ¥¥
&=¥displaystyle{f^¥prime(a)(x-a)}}¥¥ ¥¥
g(x)
&=¥displaystyle{¥left[
¥frac{dg(x)}{dx}¥right]_{x=a}(x-a)} ¥¥ ¥¥
&=¥displaystyle{g^¥prime(a)(x-a)}}
¥end{array}¥right.

〈2-32〉
¥lim_{x¥rightarrow a}¥frac{f(x)}{g(x)}
=¥lim_{x¥rightarrow }
¥frac{f^¥prime(a)(x-a)}{g^¥prime(a)(x-a)}
=¥frac{f^¥prime(a) }{g^¥prime(a)}

〈2-33〉
A=¥lim_{x¥rightarrow 0}¥frac{¥sin(x)}{x}

〈2-34〉
A=¥lim_{x¥rightarrow 0}¥frac{¥cos(x)}{1}=1

〈2-35〉
f(x)=¥frac{x}{e^{ax}-1}

〈2-36〉
¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{dx}{dt}=1}¥¥ ¥¥
¥displaystyle{¥frac{
d(e^{ax}-1)}{dx}=¥frac{de^{ax}}{dx}=ae^{ax}
¥end{array}¥right.

〈2-37〉
¥lim_{x¥rightarrow 0}¥frac{x}{e^{ax}-1}=
¥lim_{x¥rightarrow 0}¥frac{1}{ae^{ax}}=¥frac{1}{a}

〈2-38〉 (2.4.1)
x(t)=x_0+¥int_{t_0}^{t}v(t)dt

〈2-39〉 (2.4.2)
v(t)=v_0+¥int_{t_0}^{t}a(t)dt

〈3-1〉 (3.1.1)
z=f(x,y)

〈3-2〉 (3.1.2)
P=¥frac{nRT}{V}

〈3-3〉 (3.1.3)
¥lim_{h¥rightarrow0}¥frac{f(x+h,y)-f(x,y)}{h}
¥equiv¥frac{¥partial f}{¥partial x}

〈3-4〉 (3.1.4)
¥lim_{h¥rightarrow0}¥frac{f(x,y+h)-f(x,y)}{h}
¥equiv¥frac{¥partial f}{¥partial y}

〈3-5〉 (3.1.5)
¥frac{¥partial}{¥partial x}
¥left(¥frac{¥partial f(x,y)}{¥partial x}¥right)
=¥frac{¥partial^2f(x,y)}{¥partial x^2}=f_{xx}(x,y)

〈3-6〉 (3.1.6)
¥frac{¥partial}{¥partial y}
¥left(¥frac{¥partial f(x,y)}{¥partial x}¥right)
=¥frac{¥partial^2f(x,y)}{ ¥partial y¥partial x}
=f_{xy}(x,y)

〈3-7〉 (3.1.7)
f(x,y) =x^2y+xy^2+y^3

〈3-8〉 (3.1.8)
¥begin{array}{l}
¥left¥{¥begin{array}{l}
f_x(x,y)=2xy+y^2 ¥¥ ¥¥
f_y(x,y)=x^2+2xy+3y^2
¥end{array}¥right. ¥¥ ¥¥
¥left¥{¥begin{array}{l}
f_{xx}(x,y)=2y ¥¥ ¥¥

f_{xy}(x,y)=2x+2y ¥¥ ¥¥
f_{yx}(x,y)=2x+2y ¥¥ ¥¥
f_{yy}(x,y)=2x+6y
¥end{array}¥right.
¥end{array}

〈3-9〉 (3.1.9)
¥Delta f(x,y) =f(x+¥Delta x, y+¥Delta y)-f(x,y)

〈3-10〉
¥Delta f(x,y)=
¥left¥{¥frac{f(x+¥Delta x, y+¥Delta y)
-f(x, y+¥Delta y)}
{¥Delta x}¥right¥}¥Delta x
+¥left¥{¥frac{f(x, y+¥Delta y)
-f(x, y)}{¥Delta y}¥right¥}¥Delta y

〈3-11〉
¥begin{array}{rl}
¥Delta f(x,y)
&¥rightarrow
¥displaystyle{¥left¥{¥frac{f(x+¥Delta x, y)-
f(x, y)}{¥Delta x}¥right¥}¥Delta x}
+¥displaystyle{¥left¥{¥frac{f(x, y+¥Delta y)-
f(x, y)}{¥Delta y}¥right¥}¥Delta y} ¥¥ ¥¥
&¥rightarrow
¥displaystyle{
¥frac{¥partial f(x, y)}{¥partial x}¥Delta x+
¥frac{¥partial f(x, y)}{¥partial y}¥Delta y} ¥¥ ¥¥
&=f_x(x, y)¥Delta x+ f_y(x, y)¥Delta y
¥end{array}

〈3-12〉 (3.1.10)
df(x,y)= ¥frac{¥partial f(x, y)}{¥partial x}dx+
¥frac{¥partial f(x, y)}{¥partial y}dy

〈3-13〉 (3.1.11)
¥begin{array}{l}
¥displaystyle{¥frac{¥partial P(x, y)}{¥partial y}=
¥frac{¥partial Q(x, y)}{¥partial x} ¥¥ ¥¥

```
¥quad¥mbox{namely}¥quad P_y(x, y)=Q_x(x, y)
¥end{array}
```

〈3-14〉 (3.1.12)

```
¥left¥{¥begin{array}{l}
¥displaystyle{
¥frac{¥partial f(x, y)}{¥partial x}=P(x,y)} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial f(x, y)}{¥partial y}=Q(x,y)}
¥end{array}¥right.
```

〈3-15〉 (3.1.13)

```
df(x, y)=P(x,y)dx+Q(x,y)dy
```

〈3-16〉

```
¥frac{¥partial P}{¥partial y}
=¥frac{¥partial¥left(¥displaystyle{
¥frac{¥partial f}{¥partial x}}¥right)}{¥partial y}=
¥frac{¥partial^2f}{¥partial y¥partial x}=
¥frac{¥partial¥left(¥displaystyle{
¥frac{¥partial f}{¥partial y}}¥right)}{¥partial x}=
¥frac{¥partial Q}{¥partial x}
```

〈3-17〉

```
¥mbox{(a)}¥quad(3x^2+2xy-2y^2)dx+(x^2-4xy)dy
```

〈3-18〉

```
¥left¥{¥begin{array}{l}
¥displaystyle{
¥frac{¥partial P}{¥partial y}=2x-4y} ¥¥ ¥¥
¥displaystyle{¥frac{
¥partial Q}{¥partial x}=2x-4y}
¥end{array}¥right.,¥quad¥mbox{therefore}¥quad
¥frac{¥partial P}{¥partial y}
=¥frac{¥partial Q}{¥partial x}
```

〈3-19〉

```
df(x, y)=P(x,y)dx+Q(x,y)dy
```

〈3-20〉

```
¥begin{array}{l}
¥displaystyle{¥frac{¥partial f}{¥partial x}}=
P(x,y)=3x^2+2xy-2y^2} ¥¥ ¥¥
¥displaystyle{¥frac{¥partial f}{¥partial y}}=
Q(x,y)=x^2-4xy}
¥end{array}

＜3-21＞
f(x, y)=x^3+x^2y-2xy^2+C

＜3-22＞ (3.1.14)
¥left¥{¥begin{array}{rl}
d¥left[f(x)g(x)¥right]&=g(x)df(x)+f(x)dg(x) ¥¥ ¥¥
&=¥displaystyle{
¥left(g¥frac{df}{dx}+f¥frac{dg}{dx}¥right)dx
¥end{array}

＜3-23＞
¥lim_{¥Delta x¥rightarrow0}¥left[
¥frac{z(x+¥Delta x)-z(x)}{¥Delta x}¥right]
=¥frac{dz}{dx}

＜3-24＞
¥lim_{¥Delta t¥rightarrow0}¥left[
¥frac{x(t+¥Delta t)-x(t)}{¥Delta t}¥right]
=¥frac{dx}{dt}

＜3-25＞ (3.1.15)
¥frac{dz}{dt}=¥frac{dz}{dx}¥frac{dx}{dt}

＜3-26＞ (3.1.16)
dz=¥frac{df(x)}{dx}dx=¥frac{df}{dx}¥frac{dg}{dt}dt

＜3-27＞
dz=¥left(¥frac{¥partial z}{¥partial x}¥frac{dx}{dt}
+¥frac{¥partial z}{¥partial y}¥frac{dy}{dt}¥right)dt

＜3-28＞ (3.1.17)
¥frac{dz}{dt}=¥frac{¥partial z}{¥partial x}
¥frac{dx}{dt}+¥frac{¥partial z}{¥partial y
```

¥frac{dy}{dt}

<3-29> df(x)=¥frac{df(x)}{dx}dx

<3-30> dg(x,y)=¥frac{¥partial g(x,y)}{¥partial x}dx
+¥frac{¥partial g(x,y)}{¥partial y}dy

<3-31>
dz=¥left(¥frac{¥partial z}{¥partial x}
¥frac{¥partial x}{¥partial r}
+¥frac{¥partial z}{¥partial y}
¥frac{¥partial y}{¥partial r}¥right)dr
+¥left(¥frac{¥partial z}{¥partial x}
¥frac{¥partial x}{¥partial s}
+¥frac{¥partial z}{¥partial y}
¥frac{¥partial y}{¥partial s}¥right)ds

<3-32> (3.1.18)
¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{¥partial z}{¥partial r}
=¥frac{¥partial z}{¥partial x}
¥frac{¥partial x}{¥partial r}
+¥frac{¥partial z}{¥partial y}
¥frac{¥partial y}{¥partial r}} ¥¥ ¥¥
¥displaystyle{¥frac{¥partial z}{¥partial s}
=¥frac{¥partial z}{¥partial x}
¥frac{¥partial x}{¥partial s}
+¥frac{¥partial z}{¥partial y}
¥frac{¥partial y}{¥partial s}}
¥end{array}¥right.

<3-33>
¥left¥{¥begin{array}{l}
x+ct¥equiv p(x,t) ¥¥ ¥¥
x-ct¥equiv q(x,t)
¥end{array}¥right.

<3-34> (3.1.19)
u(x,t)=f(p)+g(q)

〈3-35〉

```
¥left¥{¥begin{array}{l}
p(x,t)¥equiv x+ct ¥¥ ¥¥
q(x,t)¥equiv x-ct
¥end{array}¥right.
```

〈3-36〉

```
¥left¥{¥begin{array}{ll}
¥displaystyle{¥frac{¥partial p}{¥partial x}}=1,
& ¥quad¥displaystyle{¥frac{¥partial p}{¥partial t}}=
c
¥¥ ¥¥
¥displaystyle{¥frac{¥partial q}{¥partial x}}=1,
& ¥quad¥displaystyle{¥frac{¥partial q}{¥partial t}}=
-c
¥end{array}¥right.
```

〈3-37〉

```
¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{¥partial u}{¥partial x}
=¥frac{df}{dp}¥frac{dp}{dx}+
¥frac{dg}{dq}¥frac{¥partial q}{¥partial  x}
=¥frac{df}{dp}+¥frac{dg}{dq}} ¥¥ ¥¥
¥displaystyle{¥frac{¥partial u}{¥partial t}
=¥frac{df}{dp}¥frac{dp}{dt}+
¥frac{dg}{dq}¥frac{¥partial q}{¥partial  t}
=c¥left(¥frac{df}{dp}-¥frac{dg}{dq}¥right)}
¥end{array}¥right.
```

〈3-38〉

```
¥left¥{¥begin{array}{ll}
¥displaystyle{¥frac{¥partial^2u}{¥partial x^2}}
&=¥displaystyle{¥frac{¥partial}{¥partial x}
¥left(¥frac{df}{dp}+¥frac{dg}{dq}¥right)} ¥¥ ¥¥
&=¥displaystyle{
¥left[¥frac{d}{dp}¥left(¥frac{df}{dp}¥right)¥right]
¥frac{¥partial p}{¥partial x}+¥left[¥frac{d}{dq}
¥left(¥frac{dg}{dq}¥right)¥right]
¥frac{¥partial q}{¥partial x} ¥¥ ¥¥
&=¥displaystyle{¥frac{d^2f}{dp^2}
```

+¥frac{d^2g}{dq^2}} ¥¥ ¥¥
¥displaystyle{¥frac{¥partial^2u}{¥partial t^2}}
&=¥displaystyle{¥frac{¥partial}{¥partial t}
¥left[c¥left(¥frac{df}{dp}
-¥frac{dg}{dq}¥right)¥right]} ¥¥ ¥¥
&=¥displaystyle{c¥left(¥left[¥frac{d}{dp}
¥left(¥frac{df}{dp}¥right)¥right]
¥frac{¥partial p}{¥partial t}+
¥left[¥frac{d}{dq}¥left(¥frac{dg}{dq}¥right)¥right]
¥frac{¥partial q}{¥partial t} ¥right)¥¥ ¥¥
&=c^2¥displaystyle{¥left(¥frac{d^2f}{dp^2}
+¥frac{d^2g}{dq^2}¥right)}
¥end{array}¥right.

〈3-39〉
¥frac{¥partial^2u}{¥partial t^2}-
c^2¥frac{¥partial^2u}{¥partial x^2}=0

〈3-40〉
F(x+dx,y+dy)=F(x,y)
+¥frac{¥partial F(x,y)}{¥partial x}dx
+¥frac{¥partial F(x,y)}{¥partial y}dy

〈3-41〉 (3.1.19)
¥begin{array}{rl}
dF&¥equiv F(x+dx,y+dy)-F(x,y) ¥¥ ¥¥
&=Adx+Bdy
¥end{array}

〈3-42〉 (3.1.20)
¥left¥{¥begin{array}{l}
¥displaystyle{
¥frac{¥partial F(x,y)}{¥partial x}=A} ¥¥ ¥¥
¥displaystyle{¥frac{
¥partial F(x,y)}{¥partial y}=B}
¥end{array}¥right.

〈3-43〉 (3.1.21)
G=F(x,y)-xA

〈3-44〉 (3.1.22)
¥left¥{¥begin{array}{l}
G¥rightarrow G+dG ¥¥ ¥¥
F¥rightarrow F+dF ¥¥ ¥¥
x¥rightarrow x+dx ¥¥ ¥¥
A¥rightarrow A+dA
¥end{array}¥right.

〈3-45〉
¥begin{array}{rl}
G+dG&=F+dF-(x+dx)(A+dA) ¥¥ ¥¥
&=F+dF-(xA+xdA+Adx+dxdA)
¥end{array}¥right.

〈3-46〉
¥begin{array}{rl}
G+dG&=(F-xA)+(dF-xdA-Adx)-dxdA ¥¥ ¥¥
&=(F-xA)+(Ddy-xdA)-dxdA
¥end{array}¥right

〈3-47〉 (3.1.23)
dG=Bdy-xdA

.
〈3-48〉
¥mbox{(i)}¥quad f(x,v)=¥frac{m}{2}v^2-V(x)

〈3-49〉
¥mbox{(ii)}¥quad¥frac{d}{dt}
¥left(¥frac{¥partial f}{¥partial v}
¥right)-¥frac{¥partial f}{¥partial x}=0

〈3-50〉
¥mbox{(iii)}¥quad¥frac{d}{dt}(mv)=-¥frac{dV}{dx}

〈3-51〉
¥mbox{(iv)}¥quad
¥frac{¥partial f(x,v)}{¥partial v}=p

〈3-52〉

¥mbox{(v)}¥quad g=vp-f(x,v)

〈3-53〉
d(vp)=vdp+pdv

〈3-54〉
¥begin{array}{rl}
df&=¥displaystyle{¥frac{¥partial f}{¥partial v}dv+
¥frac{¥partial f}{¥partial x}dx} ¥¥ ¥¥
&=¥displaystyle{pdv-¥frac{dV}{dx}dx}
¥end{array}

〈3-55〉
¥begin{array}{rl}
dg&=d(vp)-df ¥¥ ¥¥
&=vdp+pdv-¥displaystyle{¥left(pdv
-¥frac{dV}{dx}dx¥right) ¥¥ ¥¥
&=vdp+¥displaystyle{¥frac{dV}{dx}dx}
¥end{array}

〈3-56〉
dg=¥frac{¥partial g}{¥partial x}dx
+¥frac{¥partial g}{¥partial p}dp

〈3-57〉
¥mbox{(vi)}¥qqad¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{¥partial g}{¥partial x}
=¥frac{dV}{dx}} ¥¥ ¥¥
¥displaystyle{¥frac{¥partial g}{¥partial p}
=v=¥frac{p}{m}}
¥end{array}¥right.

〈3-58〉
¥mbox{(vii)}¥quad¥begin{array}{rl}
g&=¥displaystyle{¥frac{p}{m}p-¥left[¥frac{m}{2}
¥left(¥frac{p}{m}¥right)^2-V(x)¥right] ¥¥ ¥¥
&=¥displaystyle{¥frac{p^2}{2m}+V(x)
¥end{array}

〈3-59〉

```
¥frac{dg}{dt}=¥frac{¥partial g}{¥partial x}
¥frac{dx}{dt}
+¥frac{¥partial g}{¥partial p}¥frac{dp}{dt}

<3-60>
¥mbox{(ix)}¥qquad¥begin{array}{rl}
¥displaystyle{¥frac{dg}{dt}}
&=¥displaystyle{¥frac{¥partial g}{¥partial x}
¥frac{¥partial g}{¥partial p}
+¥frac{¥partial g}{¥partial p}
¥left(-¥frac{¥partial g}{¥partial x}¥right)} ¥¥ ¥¥
&=0
¥end{array}

<3-61>
dU=¥left(¥frac{¥partial U}{¥partial S}¥right)_VdS
+¥left(¥frac{¥partial U}{¥partial V}¥right)_SdV

<3-62>
¥left¥{¥begin{array}{l}
¥displaystyle{T=
¥left(¥frac{¥partial U}{¥partial S}¥right)_V} ¥¥ ¥¥
¥displaystyle{
p=-¥left(¥frac{¥partial U}{¥partial V}¥right)_S
¥end{array}¥right.

<3-63>
¥mbox{(a)}¥quad dU=TdS-pdV

<3-64>
¥mbox{(b)}¥quad¥left¥{¥begin{array}{ll}
U &
(¥mbox{Internal Energy with the variables}
(S,V))¥¥ ¥¥
F=U-TS &
(¥mbox{Helmholtz Free Energy with the variables}
(T,V))¥¥ ¥¥
H=U+pV &
(¥mbox{Enthalpy with the variables}
(S,p))¥¥ ¥¥
```

```
G=H-TS &
(¥mbox{Gibbs Free Energy with the variables}
(T,p))
¥end{array}¥right.

<3-65>
¥mbox{(c)}¥quad
dF=¥frac{¥partial F}{¥partial T}dT
+¥frac{¥partial F}{¥partial V}dV

<3-66>
¥begin{array}{rl}
dF&=dU-d(TS) ¥¥ ¥¥
&=TdS-pdV-(TdS+SdT) ¥¥ ¥¥
&=-SdT-pdV
¥end{array}

<3-67>
¥mbox{(d)}¥quad¥left¥{¥begin{array}{l}
¥displaystyle{S=
-¥left(¥frac{¥partial F}{¥partial T}¥right)_V} ¥¥ ¥¥
¥displaystyle{
p=-¥left(¥frac{¥partial F}{¥partial V}¥right)_T}
¥end{array}¥right.

<3-68>
¥begin{array}{rl}
dH&=dU+d(pV) ¥¥ ¥¥
&=TdS-pdV+(pdV+Vdp) ¥¥ ¥¥
&=TdS+Vdp
¥end{array}

<3-69>
¥mbox{(e)}¥quad¥left¥{¥begin{array}{l}
¥displaystyle{T=
¥left(¥frac{¥partial H}{¥partial S}¥right)_p}¥¥ ¥¥
¥displaystyle{V=
¥left(¥frac{¥partial H}{¥partial p}¥right)_S
¥end{array}¥right.
```

```
<3-70>
¥begin{array}{rl}
dG&=dH-d(TS) ¥¥ ¥¥
&=TdS+Vdp-(SdT+TdS) ¥¥ ¥¥
&=-SdT+Vdp
¥end{array}

<3-71>
¥mbox{(f)}¥quad¥left¥{¥begin{array}{l}
¥displaystyle{
S=-¥left(¥frac{¥partial G}{¥partial T}¥right)_p}
¥¥ ¥¥
¥displaystyle{
V=¥left(¥frac{¥partial G}{¥partial p}¥right)_T
¥end{array}¥right.

<3-72> (3.2.1)
1¥times1=1

<3-73> (3.2.2)
i¥times i=-1

<3-74> (3.2.3)
1¥times a+i¥times b=a+ib¥equiv z

<3-75> (3.2.4)
z=x+iy

<3-76> (3.2.5)
¥left¥{¥begin{array}{l}
x=r¥cos¥theta ¥¥ ¥¥
y=r¥sin¥theta
¥end{array}¥right.

<3-77>
¥begin{array}{rl}
z&=r(¥cos¥theta +i¥sin¥theta) ¥¥ ¥¥
&=¥displaystyle{¥left[¥left(1-¥frac{¥theta^2}{2!}
+¥frac{¥theta^4}{4!}-¥cdots¥right)
+i¥left(¥theta-¥frac{¥theta^3}{3!}
```

+ ¥frac{¥theta^5}{5!}-¥cdots¥right)¥right]
¥end{array}

〈3-78〉
¥begin{array}{rl}
z&=¥displaystyle{r¥left[¥left(1+¥frac{(i¥theta)^2}{2
!}
+¥frac{(i¥theta)^4}{4!}+¥cdots¥right)
+¥left(i¥theta-¥frac{(i¥theta)^3}{3!}
+¥frac{(i¥theta)^5}{5!}+
¥cdots¥right)¥right] ¥¥ ¥¥
&=¥displaystyle{r¥sum_{k=0}^{¥infty}
¥frac{(i¥theta)^k}{k!}}
¥end{array}

〈3-79〉 (3.2.6)
¥begin{array}{rl}
z&=x+iy ¥¥ ¥¥
&=r(¥cos¥theta+i¥sin¥theta) ¥¥ ¥¥
&=re^{i¥theta}
¥end{array}

〈3-80〉 (3.2.7)
e^{i¥theta}=¥cos¥theta+i¥sin¥theta

〈3-81〉 (3.2.8)
¥left¥{¥begin{array}{l}
r=|z|=¥sqrt{x^2+y^2} ¥¥ ¥¥
¥displaystyle{¥tan¥theta=¥frac{y}{x}¥quad
¥mbox{or}¥quad¥theta=¥mbox{arg}(z)
=¥tan^{-1}¥left(¥frac{y}{x}¥right)
¥end{array}¥right.

〈3-82〉 (3.2.9)
¥begin{array}{l}
360^¥circ=2¥pi¥;¥mbox{[radian]} ¥¥ ¥¥
¥mbox{or}¥¥ ¥¥
180^¥circ=¥pi¥;¥mbox{[radian]}¥simeq3.14¥;
¥mbox{[radian]}
¥end{array}

〈3-83〉
¥pi/6¥simeq0.52

〈3-84〉
e^{i¥pi/6}=¥sqrt{3}/2+i(1/2)

〈3-85〉
¥pi/4¥simeq0.79

〈3-86〉
e^{i¥pi/4}=1/¥sqrt{2}+i¥left(1/¥sqrt{2}¥right)

〈3-87〉
¥pi/3¥simeq1.05

〈3-88〉
e^{i¥pi/3}=1/2+i¥left(¥sqrt{3}/2¥right)

〈3-89〉
¥pi/2¥simeq1.57

〈3-90〉
e^{i¥pi/2}=i

〈3-91〉
3¥pi/4¥simeq2.36

〈3-92〉
e^{i3¥pi/4}=-1/¥sqrt{2}+i¥left(1/¥sqrt{2}¥right)

〈3-93〉
¥pi¥simeq3.14

〈3-94〉
e^{i¥pi}=-1

〈3-95〉
5¥pi/4¥simeq3.93

〈3-96〉
e^{i5¥pi/4}=-1/¥sqrt{2}-i¥left(1/¥sqrt{2}¥right)

〈3-97〉
3¥pi/2¥simeq4.71

〈3-98〉
e^{i3¥pi/2}=-i

〈3-99〉
7¥pi/4¥simeq5.50

〈3-100〉
e^{i7¥pi/4}=1/¥sqrt{2}-i¥left(1/¥sqrt{2}¥right)

〈3-101〉
2¥pi¥simeq6.28

〈3-102〉
e^{i2¥pi}=1

〈3-103〉 (3.2.10)
¥left¥{¥begin{array}{l}
¥displaystyle{¥cos¥theta
=¥frac{1}{2}¥left(e^{i¥theta}
+e^{-i¥theta}¥right)} ¥¥ ¥¥
¥displaystyle{¥sin¥theta
=¥frac{1}{2i}¥left(e^{i¥theta}
-e^{-i¥theta}¥right)}
¥end{array}¥right.

〈3-104〉 (3.2.11)
¥left(¥cos¥theta+i¥sin¥theta¥right)^n
=¥cos(n¥theta)+i¥sin(n¥theta)

〈3-105〉
r=r_1r_2

〈3-106〉
¥theta=¥theta_1+¥theta_2¥quad¥Rightarrow

¥quad¥mbox{arg}(z)
=¥mbox{arg}(z_1)+¥mbox{arg}(z_2)

〈3-107〉 (3.2.12)
z^*=x-iy

〈3-108〉 (3.2.13)
z^*=re^{-i¥theta}

〈3-109〉 (3.2.14)
¥sqrt{zz^*}=|z|=r

〈3-110〉
¥begin{array}{rl}
¥cos z&=¥displaystyle{
¥frac{1}{2}¥left[¥left(¥cos x+i¥sin x¥right)e^{-y}
+¥left(¥cos x-i¥sin x¥right)e^y¥right]} ¥¥ ¥¥
&=¥displaystyle{
¥frac{1}{2}¥left[¥left(e^y+e^{-y}¥right)¥cos x
-i¥left(e^y-e^{-y}¥right)¥sin x¥right]}
¥end{array}

〈3-111〉
¥left¥{¥begin{array}{l}
u=¥displaystyle{
¥frac{1}{2}¥left(e^{y}+e^{-y}¥right)¥cos x} ¥¥ ¥¥
v=¥displaystyle{
-¥frac{1}{2}¥left(e^{y}-e^{-y}¥right)¥sin x}
¥end{array}¥right.

〈3-112〉
¥left¥{¥begin{array}{l}
¥displaystyle{
¥frac{1}{2}¥left(e^{y}+e^{-y}¥right)
=¥cosh y} ¥¥ ¥¥
¥displaystyle{
-¥frac{1}{2}¥left(e^{y}-e^{-y}¥right)
=¥sinh x}
¥end{array}¥right.

〈3-113〉
¥left¥{¥begin{array}{l}
u=¥cosh y¥cos x ¥¥ ¥¥
v=-¥sinh y¥sin x
¥end{array}¥right.

〈3-114〉
¥sin z=¥cosh y¥sin x+i¥sinh y¥cos x

〈3-115〉
z=-1¥quad¥mbox{or}¥quad z^2-z+1=0

〈3-116〉
z=¥frac{1¥pm¥sqrt{-3}}{2}
=¥frac{1¥pm i¥sqrt{3}}{2}

〈3-117〉
z=-1,¥quad¥frac{1+i¥sqrt{3}}{2},¥quad
¥frac{1-i¥sqrt{3}}{2}

〈3-118〉
¥left¥{¥begin{array}{l}
¥displaystyle{e^{i(¥pi/4)}
=¥cos¥left(¥frac{¥pi}{4}¥right)
+i¥sin¥left(¥frac{¥pi}{4}¥right)
=¥frac{1}{¥sqrt{2}}+¥frac{i}{¥sqrt{2}}} ¥¥ ¥¥
¥displaystyle{e^{i(3¥pi/4)}
=¥cos¥left(¥frac{3¥pi}{4}¥right)
+i¥sin¥left(¥frac{3¥pi}{4}¥right)
=-¥frac{1}{¥sqrt{2}}+¥frac{i}{¥sqrt{2}}} ¥¥ ¥¥
¥displaystyle{e^{i(5¥pi/4)}
=¥cos¥left(¥frac{5¥pi}{4}¥right)+
i¥sin¥left(¥frac{5¥pi}{4}¥right)=
-¥frac{1}{¥sqrt{2}}-¥frac{i}{¥sqrt{2}}} ¥¥ ¥¥
¥displaystyle{e^{i(7¥pi/4)}
=¥cos¥left(¥frac{7¥pi}{4}¥right)+
i¥sin¥left(¥frac{7¥pi}{4}¥right)=
¥frac{1}{¥sqrt{2}}-¥frac{i}{¥sqrt{2}}}
¥end{array}¥right.

〈3-119〉
¥begin{array}{rl}
|w|^2&=¥displaystyle{
¥frac{i+1}{¥cos¥theta-i¥sin¥theta}
¥left(¥frac{i+1}{¥cos¥theta-i¥sin¥theta }¥right)^*}
¥¥ ¥¥
&=¥displaystyle{
¥frac{(i+1)(-i+1)}{(¥cos¥theta-i¥sin¥theta)
(¥cos¥theta+i¥sin¥theta)}} ¥¥ ¥¥
&=¥displaystyle{
¥frac{2}{¥cos^2¥theta+¥sin^2¥theta}} ¥¥ ¥¥
&=2
¥end{array}

〈4-1〉 (4.1.1)
¥frac{dF(x)}{dx}=f(x)

〈4-2〉 (4.1.2)
¥frac{dF(x)}{dx}=f(x)

〈4-3〉 (4.1.3)
¥int f(x)dx

〈4-4〉
f(x)

〈4-5〉
F(x)=¥int f(x)dxdA

〈4-6〉
x^n

〈4-7〉
¥frac{x^{n+1}}{n+1}

〈4-8〉
¥frac{1}{x}

〈4-9〉

¥ln|x|

〈4-10〉
¥frac{1}{x^{n+1}}¥quad
¥mbox{(n is a natural number.)}¥quad
¥mbox{(*)}

〈4-11〉
-¥frac{1}{nx^n}

〈4-12〉
e^x

〈4-13〉
e^x

〈4-14〉
¥sin x

〈4-15〉
-¥cos x

〈4-16〉
¥cos x

〈4-17〉
¥sin x

〈4-18〉
¥sin^2x ¥quad ¥mbox{(*)}

〈4-19〉
-¥frac{1}{4}¥sin 2x+¥frac {x}{2}

〈4-20〉
¥cos^2x ¥quad ¥mbox{(*)}

〈4-21〉
¥frac{1}{4}¥sin 2x+¥frac {x}{2}

〈4-22〉
¥frac{1}{¥sqrt{a+x}}¥quad¥mbox{(a is a constant.)}
¥quad¥mbox{(*)}

〈4-23〉
2¥sqrt{a+x}

〈4-24〉
¥frac{1}{(a+x)^{3/2}}¥quad¥mbox{(a is a constant.)}
¥quad¥mbox{(*)}

〈4-25〉
-¥frac{2}{¥sqrt{a+x}}

〈4-26〉
¥sqrt{a^2+x^2}¥quad¥mbox{(a is a constant.)} ¥quad
¥mbox{(*)}

〈4-27〉
¥frac{x¥sqrt{a^2+x^2}}{2}+¥frac{a^2}{2}¥ln¥left|x
+¥sqrt{a^2+x^2}¥right|

〈4-28〉
¥sqrt{a^2-x^2}¥quad¥mbox{(a is a constant.)}
¥quad ¥mbox{(*)}

〈4-29〉
¥frac{x¥sqrt{a^2-x^2}}{2}+¥frac{a^2}{2}¥sin^{-1}
¥left(¥frac{x}{a}¥right)

〈4-30〉 (4.2.1)
F(b)-F(a)¥equiv¥int_a^b f(x)dx

〈4-31〉 (4.2.2)

〈5-1〉 (5.1.1)
¥frac{dN}{dt}=-kN

〈5-2〉 (5.1.2)
L¥frac{d^2I}{dt^2}+R¥frac{dI}{dt}+¥frac{I}{C}=0

〈5-3〉 (5.1.3)
¥alpha¥frac{d^2¥psi}{dx^2}+V(x)¥psi=E¥psi

〈5-4〉 (5.1.4)
¥frac{¥partial^2¥Phi}{¥partial x^2}+
¥frac{¥partial^2¥Phi}{¥partial y^2}
+¥frac{¥partial^2¥Phi}{¥partial z^2}=0

〈5-5〉 (5.1.5)
¥frac{¥partial¥phi}{¥partial t}=
D¥frac{¥partial^2¥phi}{¥partial x^2

〈5-6〉
¥left[¥mbox{A term proportional to }
¥frac{d^2y}{dx^2}¥right]
+¥left[¥mbox{A term proportional to }¥frac{dy}{dx}¥r
ight]
+¥left[¥mbox{A term proportional to }y¥right]
=¥left[¥mbox{A funcion $Q(x)$ independent of y}¥ri
ght]

〈5-7〉 (5.1.6)
¥frac{dy}{dx}=p(x)q(y)

〈5-8〉 (5.1.7)
¥frac{1}{q(y)}¥frac{dy}{dx}=p(x)

〈5-9〉 (5.1.8)
¥int¥frac{1}{q(y)}¥frac{dy}{dx}dx=¥int p(x)dx+C

〈5-10〉 (5.1.9)
¥int¥frac{1}{q(y)}dy=¥int p(x)dx+C

〈5-11〉
¥int¥frac{1}{y+1}dy=¥int¥frac{1}{x+1}dx+C

〈5-12〉 (5.1.10)
¥int¥frac{1}{x+a}dx=¥ln|x+a|

<5-13>
¥ln|y+1|=¥ln|x+1|+C

<5-14>
¥ln|y+1|-¥ln|x+1|=¥ln¥left|¥frac{y+1}{x+1}¥right|=C

<5-15>
¥left|¥frac{y+1}{x+1}¥right|=e^C

<5-16>
¥frac{y+1}{x+1}=C

<5-17>
y=C(x+1)-1

<5-18>
¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{dy}{dx}=C} ¥¥ ¥¥
¥displaystyle{¥frac{y+1}{x+1}=C}
¥end{array}¥right.

<5-19> (5.1.11)
¥frac{dy}{dx}+p(x)y=q(x)

<5-20> (5.1.12)
¥frac{dy}{dx}+p(x)y=0

<5-21>
¥int¥frac{1}{y}dy=
-¥int p(x)dx+C¥quad
¥mbox{(C is an integration constant.)}

<5-22> (5.1.13)
y=Ce^{¥displaystyle{-¥int p(x)dx}}

<5-23> (5.1.14)
y=C(x)e^{¥displaystyle{-¥int p(x)dx}}

<5-24> (5.1.15)
¥frac{dC(x)}{dx}=

q(x)e^{¥displaystyle{¥int p(x)dx}}

<5-25>　(5.1.16)
¥begin{array}{rl}
C(x)&=¥int X(x)dx+C ¥¥ ¥¥
&=¥int¥left¥{q(x)e^{¥displaystyle{
¥int p(x)dx}}¥right¥}dx+C
¥end{array}

<5-26>　(5.1.17)
y=e^{-¥displaystyle{¥int p(x)dx}}
¥left[¥int¥left¥{q(x)e^{¥displaystyle{
¥int p(x)dx}}¥right¥}dx+C¥right]

<5-27>　(5.1.18)
L¥frac{dI}{dt}+RI=V(t)

<5-28>　I(t)=
¥frac{1}{L}e^{-(R/L)t}
¥left[¥int¥left¥{e^{(R/L)t}V(t)¥right¥}dt
+C_1¥right]

<5-29>
I(t)=¥frac{V_0}{R}+C_2e^{-(R/L)t}

<5-30>
I(t)=¥frac{V_0}{R}¥left¥{1-e^{-(R/L)t}¥right¥}

<5-31>
e^{-(R/L)t}¥simeq 1-¥frac{R}{L}t

<5-32>
I(t)¥simeq
¥frac{V_0}{R}¥left[1-¥left¥{1-
¥frac{R}{L}t¥right¥}¥right]=¥frac{V_0}{L}t

<5-33>　(5.1.19)
¥frac{dy}{dx}=f¥left(¥frac{y}{x}¥right)

<5-34>

¥frac{y(x)}{x}=u(x)

〈5-35〉
¥frac{dy}{dx}=¥frac{d(xu)}{dx}=u+x¥frac{du}{dx}

〈5-36〉
u+x¥frac{du}{dx}=f(u)

〈5-37〉 ¥quad ¥mbox{(*)}
¥frac{du}{dx}=¥frac{f(u)-u}{x}

〈5-38〉
¥frac{dy}{dx}=¥frac{x^2+y^2}{xy}

〈5-39〉
¥begin{array}{rl}
¥frac{x^2+y^2}{xy}&=¥frac{x}{y}+¥frac{x}{y} ¥¥ ¥¥
&=¥left(¥frac{y}{x}¥right)^{-1}+¥frac{x}{y}
¥end{array}

〈5-40〉
¥frac{du}{dx}=¥frac{1/u}{x}

〈5-41〉
u=¥sqrt{¥ln(x^2)+C}

〈5-43〉 (5.1.20)
¥frac{dy}{dx}=-¥frac{p(x,y)}{q(x,y)}

〈5-44〉 (5.1.21)
¥frac{¥partial p}{¥partial y}
=¥frac{¥partial q}{¥partial x}

〈5-45〉
¥mbox{(I)}¥quad¥frac{¥partial P}{¥partial y}
=¥frac{¥partial Q}{¥partial x}

〈5-46〉
¥mbox{(II)}¥quad¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{¥partial f}{¥partial x}}

=P(x,y) ¥¥ ¥¥
¥displaystyle{¥frac{¥partial f}{¥partial y}}
=Q(x,y)
¥end{array}¥right.rtial f}{¥partial y}}
=Q(x,y)
¥end{array}¥right.

〈5-47〉
¥mbox{(III)}¥quad df(x,y)=P(x,y)dx+Q(x,y)dy

〈5-48〉　(5.1.22)
p(x,y)dx+q(x,y)dy=0

〈5-49〉　(5.1.23)
du(x,y)=p(x,y)dx+q(x,y)dy

〈5-50〉　(5.1.24)
¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{¥partial u}{¥partial x}}
=p(x,y) ¥¥ ¥¥
¥displaystyle{¥frac{¥partial u}{¥partial y}}
=q(x,y)
¥end{array}¥right.

〈5-51〉
¥frac{dy}{dx}=-¥frac{x+y+1}{x-y^2+3}

〈5-52〉
¥frac{¥partial p(x,y)}{¥partial y}=
¥frac{¥partial q(x,y)}{¥partial x}=1

〈5-53〉
¥left¥{¥begin{array}{rl}
¥displaystyle{¥frac{¥partial u(x,y)}{¥partial x}}
&=p(x,y) ¥¥ ¥¥
&=x+y+1 ¥¥ ¥¥
¥displaystyle{¥frac{¥partial u(x,y)}{¥partial y}}
&=q(x,y) ¥¥ ¥¥
&=x-y^2+3
¥end{array}¥right.

〈5-54〉
u=¥frac{x^2}{2}+xy+x+g(y)

〈5-55〉
¥frac{¥partial u}{¥partial y}=x+¥frac{dg}{dy}

〈5-56〉
¥frac{dg}{dy}=-y^2+3

〈5-57〉
g=-¥frac{y^3}{3}+3y+A

〈5-58〉
u=¥frac{x^2}{2}+xy-¥frac{y^3}{3}+x+3y+A

〈5-59〉
¥frac{x^2}{2}+xy-¥frac{y^3}{3}+x+3y=C

〈5-60〉 (5.1.25)
¥frac{dy}{dx}+p(x)y=q(x)y^n ¥quad (n¥ne0,1)

〈5-61〉 (5.1.26)
¥frac{d^2y}{dx^2}+p(x)y^2+q(x)y+r(x)=0

〈5-62〉
¥frac{dx}{dt}=
k¥left(a-¥frac{x}{2}¥right)
¥left(b-¥frac{x}{2}¥right)

〈5-63〉
¥mbox{(1)}¥quad
¥int¥frac{1}{¥left(a-
¥displaystyle{¥frac{x}{2}}¥right)
¥left(b-¥displaystyle{¥frac{x}{2}}¥right)}dx
=k¥int dt+C

〈5-64〉
¥frac{1}{¥left(a-¥displaystyle{¥frac{x}{2}}¥right)
¥left(b-¥displaystyle{¥frac{x}{2}}¥right)}

=¥frac{1}{a-b}¥left[¥frac{1}{b-
¥displaystyle{¥frac{x}{2}}}-
¥frac{1}{a-¥displaystyle{¥frac{x}{2}}}¥right]

＜5-65＞
¥mbox{(2)}¥quad¥frac{1}{a-b}
¥int¥frac{1}{b-¥displaystyle{
¥frac{x}{2}}}dx-¥frac{1}{a-b}¥int¥frac{1}{a-
¥displaystyle{¥frac{x}{2}}}dx=kt+C

＜5-66＞
¥int¥frac{1}{a-¥displaystyle{¥frac{x}{2}}}dx
=-2¥ln(x-2a)

＜5-67＞
¥ln¥frac{x-2a}{x-2b}=-¥frac{b-a}{2}(kt+C)

＜5-68＞
¥mbox{(3)}¥quad
x=2ab¥left[¥frac{1-e^{(b-a)kt/2}}{a-be^{(b-a)kt/2}}¥
right]

＜5-69＞
e^y¥simeq 1+y

＜5-70＞
A(¥mbox{radio nucleide})¥rightarrow
B(¥mbox{radio nucleide})¥rightarrow
C(¥mbox{stable nucleide})

＜5-71＞
¥Delta N_{A}(t)=-¥lambda_{A}N_{A}(t)¥Delta t

＜5-72＞
N_{A}(t)-¥lambda_{A}N_{A}(t)¥Delta t

＜5-73＞
N_{B}(t)-¥lambda_{B}N_{B}(t)¥Delta t
+¥lambda_{A}N_{A}(t)¥Delta t

<5-74>
N_{C}(t)+¥lambda_{B}N_{B}(t)¥Delta t

<5-75>
¥left¥{¥begin{array}{l}
¥mbox{(1)}¥quad N_{A}(t+¥Delta t)=
N_{A}(t)-¥lambda_{A}N_{A}(t)¥Delta t ¥¥ ¥¥
¥mbox{(2)}¥quad N_{B}(t+¥Delta t)=
N_{B}(t)-¥lambda_{B}N_{B}(t)¥Delta t+
¥lambda_{A}N_{A}(t)¥Delta t ¥¥ ¥¥
¥mbox{(3)}¥quad N_{C}(t+¥Delta t)=
N_{C}(t)+¥lambda_{B}N_{B}(t)¥Delta t
¥end{array}¥right.

<5-76>
¥frac{N_A(t+¥Delta t)-N_A(t)}{¥Delta t}

<5-77>
¥frac{N_B(t+¥Delta t)-N_B(t)}{¥Delta t}

<5-78>
¥frac{N_C(t+¥Delta t)-N_C(t)}{¥Delta t}

<5-79>
¥left¥{¥begin{array}{l}
¥mbox{(4)}¥quad ¥displaystyle{
¥frac{N_{A}(t)}{dt}}=-¥lambda_{A}N_{A}(t) ¥¥ ¥¥
¥mbox{(5)}¥quad ¥displaystyle{
¥frac{N_{B}(t)}{dt}}=-¥lambda_{B}N_{B}(t)
+¥lambda_{A}N_{A}(t) ¥¥ ¥¥
¥mbox{(6)}¥quad ¥displaystyle{
¥frac{N_{C}(t)}{dt}}=¥lambda_{B}N_{B}(t)
¥end{array}¥right.

<5-80>
¥mbox{(7)}¥quad N_{A}(t)=C_Ae^{-¥lambda_{A}t}

<5-81>
¥mbox{(8)}¥quad N_{A}(t)=N_0e^{-¥lambda_{A}t}

〈5-82〉
¥mbox{(9)}¥quad ¥frac{N_{B}(t)}{dt}
+¥lambda_{B}N_{B}(t)
=¥lambda_{A}N_0e^{-¥lambda_{A}t}

〈5-83〉
¥mbox{(10)}¥quad ¥frac{N_{B}(t)}{dt}
+¥lambda_{B}N_{B}(t)=0

〈5-84〉
¥mbox{(11)}¥quad N_{B}(t)
=C_Be^{-¥lambda_{B}t}

〈5-85〉
¥mbox{(12)}¥quad N_{B}(t)
=C_B(t)e^{-¥lambda_{B}t}

〈5-86〉
¥frac{dN_{B}(t)}{dt}=¥frac{dC_{B}(t)}{dt}
e^{-¥lambda_{B}t}
-¥lambda_{B}C_{B}(t)e^{-¥lambda_{B}t}

〈5-87〉
¥frac{dC_{B}(t)}{dt}=
¥lambda_{A}N_{0}e^{(¥lambda_{B}-¥lambda_{A})t}

〈5-88〉
C_{B}(t)=¥frac{¥lambda_{A}N_{0}}
{¥lambda_{B}-¥lambda_{A}}
¥left[e^{(¥lambda_{B}-¥lambda_{A})t}-1¥right]

〈5-89〉
N_{B}(t)=¥frac{¥lambda_{A}N_{0}}
{¥lambda_{B}-¥lambda_{A}}
¥left[e^{-¥lambda_{A}t}}-e^{-¥lambda_{B}t}}¥right]

〈5-90〉
¥frac{dN_{C}(t)}{dt}=
¥frac{¥lambda_{A}¥lambda_{B}N_{0}}
{¥lambda_{B}-¥lambda_{A}}

¥left[e^{-¥lambda_{A}t}-
e^{-¥lambda_{B}t}}¥right]

〈5-91〉
¥int e^{-at}dt=-¥frac{e^{-at}}{a}

〈5-92〉
N_{C}(t)=
¥frac{¥lambda_{A}¥lambda_{B}N_{0}}
{¥lambda_{B}-¥lambda_{A}}
¥left[-¥frac{e^{-¥lambda_{A}t}}{¥lambda_{A}}
+¥frac{e^{-¥lambda_{B}t}}{¥lambda_{B}}¥right]
+C_C

〈5-93〉
N_{C}(t)=¥frac{N_{0}}{¥lambda_{A}-¥lambda_{B}}
¥left[¥lambda_{A}(1-e^{-¥lambda_{B}t})
-¥lambda_{B}(1-e^{-¥lambda_{A}t})¥right]

〈5-94〉
N_{A}(t)=N_{0}e^{-¥lambda_{A}T}=¥frac{N_{0}}{2}

〈5-95〉
T=¥frac{¥ln 2}{¥lambda_{A}}¥simeq
¥frac{0.69}{¥lambda_{A}}

〈5-96〉 (5.1.27)
¥frac{d^2y}{dx^2}+p(x)¥frac{dy}{dx}+q(x)y=r(x)

〈5-97〉 (5.1.28)
¥frac{d^2y}{dx^2}+p(x)¥frac{dy}{dx}+q(x)y=0

〈5-98〉 (5.1.29)
¥left[¥frac{d^2}{dx^2}+p(x)¥frac{d}{dx}
+q(x)¥right]y¥equiv L(y)=0

〈5-99〉 (5.1.30)
L(y_{1})=0¥quad¥mbox{and}¥quad L(y_{2})=0

〈5-100〉 (5.1.31)

L(C_{1}y_{1}+C_{2}y_{2})
=¥frac{d^2(C_{1}y_{1}+C_{2}y_{2})}{dx^2}+
p(x)¥frac{d(C_{1}y_{1}+C_{2}y_{2})}{dx}
+q(x)(C_{1}y_{1}+C_{2}y_{2})
=C_{1}¥left[¥frac{d^2y_{1}}{dx^2}
+p(x)¥frac{dy_{1}}{dx}+q(x)y_{1}¥right]
+C_{2}¥left[¥frac{d^2y_{2}}{dx^2}
+p(x)¥frac{dy_{2}}{dx}+q(x)y_{2}¥right]
=C_{1}L(y_{1})+C_{2}L(y_{2})
=0

<5-101> (6.1.32)
C_{1}y_{1}+C_{2}y_{2}=0

<5-102> (5.1.33)
W(x)=¥left|¥begin{array}{cc}
y_{1}(x) & y_{2}(x) ¥¥ ¥¥
y'_{1}(x) & y'_{2}(x)
¥end{array}¥right|
=y_{1}(x)y'_{2}(x)-y'_{1}(x)y_{2}(x)

<5-103> (5.1.34)
¥left¥{¥begin{array}{l}
¥mbox{if }W(x)¥ne0,
¥mbox{ y_1 and y_2 are linear independent.}
¥¥ ¥¥
¥mbox{if }W(x)=0,
¥mbox{ y_1 and y_2 are linear dependent.}
¥end{array}¥right.

<5-104> (5.1.35)
¥frac{d^2y}{dx^2}+p¥frac{dy}{dx}+qy=r(x)

<5-105> (5.1.36)
¥frac{d^2y}{dx^2}+p¥frac{dy}{dx}+qy=0

<5-106> (5.1.37)
¥lambda^2+p¥lambda+q=0

<5-107>

D^2y+pDy+qy=(D^2+pD+q)y(x)=0

<5-108>
(¥lambda^2+p¥lambda+q)y(x)=0

<5-109>
¥left¥{¥begin{array}{l}
¥displaystyle{¥lambda_1=
¥frac{1}{2}¥left(-p+¥sqrt{p^2-4q}¥right)} ¥¥ ¥¥
¥displaystyle{¥lambda_2
=¥frac{1}{2}¥left(-p-¥sqrt{p^2-4q}¥right)}
¥end{array}¥right.

<5-110>
y=C_1e^{¥lambda_1x}+C_2e^{¥lambda_2x}

<5-111>
¥begin{array}{l}
y=C_1y_1(x)+C_2y_(2) ¥¥ ¥¥
¥mbox{where}¥quad ¥left¥{¥begin{array}{l}
y_1(x)=e^{¥lambda_1x} ¥¥ ¥¥
y_2(x)=e^{¥lambda_2x}
¥end{array}¥right.
¥end{array}right.
¥end{array}

<5-112>
¥begin{array}{l}
¥left¥{¥begin{array}{l}
¥displaystyle{¥lambda_1=
-¥frac{1}{2}+i¥gamma} ¥¥ ¥¥
¥displaystyle{¥lambda_2=-¥frac{1}{2}-i¥gamma}
¥end{array}¥right. ¥¥ ¥¥
¥mbox{where}¥quad¥gamma=
¥displaystyle{¥frac{¥sqrt{4q-p^2}}{2}}
¥end{array}¥begin{array}{l}
¥left¥{¥begin{array}{l}
¥displaystyle{¥lambda_1=
-¥frac{1}{2}+i¥gamma} ¥¥ ¥¥
¥displaystyle{¥lambda_2=

-¥frac{1}{2}-i¥gamma}
¥end{array}¥right. ¥¥ ¥¥
¥mbox{where}¥quad¥gamma=
¥displaystyle{¥frac{¥sqrt{4q-p^2}}{2}}
¥end{array}

＜5-113＞
¥begin{array}{rl}
y&=C_1e^{(-p/2+i¥gamma)x}+
C_2e^{(-p/2-i¥gamma)x} ¥¥ ¥¥
&=e^{(-p/2)x}¥left[C'_1¥cos(¥gamma x)+
C'_2¥sin(¥gamma x)¥right]
¥end{array}

＜5-114＞
¥begin{array}{l}
y=C_1y_1(x)+C_2y_(2) ¥¥ ¥¥
¥mbox{where}¥quad ¥left¥{¥begin{array}{l}
y_1(x)=e^{(-p/2)x}¥cos(¥gamma x) ¥¥ ¥¥
y_2(x)=e^{(-p/2)x}¥sin(¥gamma x)
¥end{array}¥right.
¥end{array}

＜5-115＞
y_1=e^{(-p/2)x}

＜5-116＞
y_2(x)=C(x)e^{(-p/2)x}

＜5-117＞
¥frac{d^2C(X)}{dx^2}=0

＜5-118＞
C(x)=C'_1+C'_2x

＜5-119＞
y_2=C'2xe^{-(p/2)x}

＜5-120＞
y=(C_1+C_2x)e^{-(p/2)x}

〈5-121〉
¥begin{array}{l}
y=C_1y_1(x)+C_2y_(2) ¥¥ ¥¥
¥mbox{where}¥quad ¥left¥{¥begin{array}{l}
y_1(x)=e^{(-p/2)x} ¥¥ ¥¥
y_2(x)=xe^{(-p/2)x}
¥end{array}¥right.
¥end{array}

〈5-122〉 (5.1.38)
y(x)=C_1y_1(x)+C_2y_2(x)

〈5-123〉 (5.1.39)
y(x)=C_1(x)y_1(x)+C_2(x)y_2(x)

〈5-124〉 (5.1.40)
¥frac{dC_1(x)}{dx}y_1(x)+
¥frac{dC_2(x)}{dx}y_2(x)=0

〈5-125〉 (5.1.41)
¥frac{dC_1(x)}{dx}¥frac{dy_1(x)}{dx}+
¥frac{dC_2(x)}{dx}¥frac{dy_2(x)}{dx}=r(x)

〈5-126〉 (5.1.52)
¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{dC_1(x)}{dx}=
-¥frac{y_2(x)r(x)}{W(x)}} ¥¥ ¥¥
¥displaystyle{¥frac{dC_2(x)}{dx}=
¥frac{y_1(x)r(x)}{W(x)}}
¥end{array}¥right. ¥¥ ¥¥
¥mbox{where}¥quad W(x)=
y_1(x)y'_2(x)-y'_1(x)y_2(x)

〈5-127〉 (5.1.43)
¥left¥{¥begin{array}{l}
C_1(x)=C''_1-¥displaystyle{
¥int¥frac{r(x)y_2(x)}{W(x)}dx} ¥¥ ¥¥
C_2(x)=C''_2+¥displaystyle{
¥int¥frac{r(x)y_1(x)}{W(x)}dx}

¥end{array}¥right.

<5-128>　(5.1.44)
¥begin{array}{l}
y_1(x)=¥displaystyle{¥left[C''_1-
¥int¥frac{r(x)y_2(x)}{W(x)}dx}¥right]y_1(x)}
+¥displaystyle{¥left[C''_2+
¥int¥frac{r(x)y_1(x)}{W(x)}dx¥right]y_2(x)}y_2(x)}
¥¥ ¥¥
=¥displaystyle{C''_1y_1(x)+C''_2y_2(x)
-y_1(x)¥left(¥int¥frac{r(x)y_2(x)}{W(x)}dx}¥right)
+y_2(x)¥left(¥int¥frac{r(x)y_1(x)}{W(x)}dx}¥right)}
¥end{array}

<5-129>　(5.2.1)
¥frac{d^2x}{dt^2}=-¥frac{g}{l}x

<5-130>　(5.2.2)
¥frac{d^2x}{dt^2}+¥omega^2x=0

<5-131>　(5.2.3)
¥lambda^2+¥omega^2=0

<5-132>　(5.2.4)
x(t)=C_1e^{i¥omega t}+C_2e^{-i¥omega t}

<5-133>
¥frac{dx}{dt}=i¥omega(C_1e^{i¥omega t}-
C_2e^{-i¥omega t})

<5-134>　(5.2.5)
¥left¥{¥begin{array}{l}
C_1+C_2=0 ¥¥ ¥¥
i¥omega(C_1-C_2)=¥omega_0
¥end{array}¥right.

<5-135>
¥left¥{¥begin{array}{l}
¥displaystyle{C_1=¥frac{¥omega_0}{2i¥omega}} ¥¥ ¥¥
¥displaystyle{C_2=-¥frac{¥omega_0}{2i¥omega}}

¥end{array}¥right.

〈5-136〉　(5.2.6)
x(t)=¥frac{¥omega_0}{¥omega}¥sin(¥omega t)

〈5-137〉　(5.2.7)
¥frac{d^2x}{dt^2}+2¥gamma¥frac{dx}{dt}+¥omega_0^2x
=0

〈5-138〉　(5.2.8)
¥lambda^2+2¥gamma¥lambda+¥omega_0^2=0

〈5-139〉
¥left¥{¥begin{array}{l}
¥lambda_1=-¥gamma+¥sqrt{¥gamma^2-¥omega_0^2} ¥¥ ¥¥
¥lambda_2=-¥gamma-¥sqrt{¥gamma^2-¥omega_0^2}
¥end{array}¥right.

〈5-140〉　(5.2.9)
x(t)=e^{-¥gamma t}¥left[C_1e^{¥sqrt{¥gamma^2-
¥omega_0^2}}+C_2e^{-¥sqrt{¥gamma^2-¥omega_0^2}}
¥right]

〈5-141〉　(5.2.10)
¥begin{array}{l}
¥displaystyle{¥frac{d^2x}{dt^2}+2¥gamma¥frac{dx}{dt}
+¥omega_0^2x=f¥cos(¥omega t)}, ¥¥ ¥¥
¥mbox{(where$¥quad¥omega_0¥ge¥gamma$.)}

〈5-142〉
¥frac{d^2x_0}{dt^2}+2¥gamma¥frac{dx_0}{dt}
+¥omega_0^2x_0=f¥cos(¥omega t)

〈5-143〉
¥frac{d^2F}{dt^2}+2¥gamma¥frac{dF}{dt}
+¥omega_0^2F

〈5-144〉
¥frac{d(f+g)}{dt}=¥frac{df}{dt}+¥frac{dg}{dt}

〈5-145〉
¥begin{array}{l}
¥displaystyle{¥frac{d^2F}{dt^2}
+2¥gamma¥frac{dF}{dt}+¥omega_0^2F} ¥¥ ¥¥
=¥displaystyle{¥left[¥frac{d^2x}{dt^2}
+2¥gamma¥frac{dx}{dt}+¥omega_0^2x¥right]
-¥left[¥frac{d^2x_0}{dt^2}+
2¥gamma¥frac{dx_0}{dt}+¥omega_0^2x_0¥right]}=0
¥end{array}

〈5-146〉　(5.2.11)
F(t)=e^{-¥gamma t}
¥left[C_1e^{¥sqrt{¥gamma^2-¥omega_0^2}t}
+C_2e^{-¥sqrt{¥gamma^2-¥omega_0^2}t}¥right]

〈5-147〉　(5.2.12)
¥frac{d^2z}{dt^2}+2¥gamma¥frac{dz}{dt}+
¥omega_0^2z=fe^{i¥omega t}

〈5-148〉
¥left¥{¥begin{array}{l}
z=x+iy ¥¥ ¥¥
¥displaystyle{¥frac{dz}{dt}=¥frac{dx}{dt}+
i¥frac{dy}{dt}} ¥¥ ¥¥
¥displaystyle{¥frac{d^2z}{dt^2}=
¥frac{d^2x}{dt^2}+i¥frac{d^2y}{dt^2}} ¥¥ ¥¥
¥displaystyle{e^{i¥omega t}=¥cos(¥omega t)+
i¥sin(¥omega t)}
¥end{array}

〈5-149〉
¥frac{de^{i¥omega t}}{dt}=i¥omega e^{i¥omega t}

〈5-150〉
¥frac{d^2e^{i¥omega t}}{dt^2}=-¥omega^2 e^{i¥omega t
}

〈5-151〉　(5.2.13)
A=¥frac{f}{¥omega_0^2-¥omega^2+i¥omega¥gamma}

<5-152>　(5.2.14)
¥left¥{¥begin{array}{l}
a=¥displaystyle{¥frac{f}{¥sqrt{(¥omega_0^2-¥omega^2)
^2
+4¥gamma^2¥omega^2}}} ¥¥ ¥¥
¥tan¥phi=¥displaystyle{¥frac{2¥gamma¥omega}{¥omega_0
^2
-¥omega^2}}¥quad¥mbox{or}¥quad
¥displaystyle{¥phi=¥tan^{-1}
¥left(¥frac{2¥gamma¥omega}{¥omega_0^2-¥omega^2}¥righ
t)}
¥end{array}¥right.

<5-153>　(5.2.15)
z(t)=ae^{i(¥omega t-¥phi)}

<5-154>　(5.2.16)
x(t)=a¥cos(¥omega t-¥phi)

<5-155>　(5.2.17)
y(t)=e^{-¥gamma t}¥left[C_1e^{¥sqrt{¥gamma^2-¥omega_
0^2}t}
+C_2e^{-¥sqrt{¥gamma^2-¥omega_0^2}t}¥right]+
a¥cos(¥omega t-¥phi)

<6-1>　(6.1.1)
¥left(¥begin{array}{ccc}
1 & 5 & 2 ¥¥ ¥¥ 4 & 2 & 3
¥end{array}¥right)

<6-2>　(6.1.2)
¥left(¥begin{array}{ccc}
a_{11} & a_{12} & a_{13} ¥¥ ¥¥ a_{21} & a_{22} & a_{
23}
¥end{array}¥right)¥pm¥left(¥begin{array}{ccc}
b_{11} & b_{12} & b_{13} ¥¥ ¥¥ b_{21} & b_{22} & b_{
23}
¥end{array}¥right)=¥left(¥begin{array}{ccc}
a_{11}¥pm b_{11} & a_{12}¥pm b_{12} & a_{13}¥pm b_{1
3}

¥¥ ¥¥
a_{21}¥pm b_{21} & a_{22}¥pm b_{22} & a_{23}¥pm b_{2
3}
¥end{array}¥right)

〈6-3〉 (6.1.3)
k¥left(¥begin{array}{cc}
a_{11} & a_{12} ¥¥ ¥¥ a_{21} & a_{22}
¥end{array}¥right)=¥left(¥begin{array}{cc}
ka_{11} & ka_{12} ¥¥ ¥¥ ka_{21} & ka_{22} }
¥end{array}¥right)

〈6-4〉
A=¥left(¥begin{array}{cccc}
a_{11} & a_{12} & ¥cdots & a_{1L} ¥¥ ¥¥
a_{21} & a_{22} & ¥cdots & a_{2L} ¥¥ ¥¥
¥cdots & ¥cdots & ¥cdots & ¥cdots ¥¥ ¥¥
a_{N1} & a_{N2} & ¥cdots & a_{NL}
¥end{array}¥right)

〈6-5〉
B=¥left(¥begin{array}{cccc}
b_{11} & b_{12} & ¥cdots & b_{1M} ¥¥ ¥¥
b_{21} & b_{22} & ¥cdots & b_{2M} ¥¥ ¥¥
¥cdots & ¥cdots & ¥cdots & ¥cdots ¥¥ ¥¥
b_{L1} & b_{L2} & ¥cdots & b_{LM}
¥end{array}¥right)

〈6-6〉
C=¥left(¥begin{array}{cccc}
c_{11} & c_{12} & ¥cdots & c_{1M} ¥¥ ¥¥
c_{21} & c_{22} & ¥cdots & c_{2M} ¥¥ ¥¥
¥cdots & ¥cdots & ¥cdots & ¥cdots ¥¥ ¥¥
c_{N1} & c_{N2} & ¥cdots & c_{NM}
¥end{array}¥right)

〈6-7〉 (6.1.4)
¥begin{array}{l}
¥left¥{¥begin{array}{l}
c_{11}=a_{11}b_{11}+¥cdots+a_{1L}b_{L1} ¥¥ ¥¥

```
c_{12}=a_{11}b_{12}+¥cdots+a_{1L}b_{L2} ¥¥ ¥¥
¥quad ¥cdots ¥¥ ¥¥
c_{1M}=a_{11}b_{1M}+¥cdots+a_{1L}b_{LM}
¥end{array}¥right. ¥¥ ¥¥
¥left¥{¥begin{array}{l}
c_{21}=a_{21}b_{11}+¥cdots+a_{2L}b_{L1} ¥¥ ¥¥
c_{22}=a_{21}b_{12}+¥cdots+a_{2L}b_{L2} ¥¥ ¥¥
¥quad ¥cdots ¥¥ ¥¥
c_{2M}=a_{21}b_{1M}+¥cdots+a_{2L}b_{LM}
¥end{array}¥right. ¥¥ ¥¥
¥left¥{¥begin{array}{l}
c_{N1}=a_{N1}b_{11}+¥cdots+a_{NL}b_{L1} ¥¥ ¥¥
c_{N2}=a_{N1}b_{12}+¥cdots+a_{NL}b_{L2} ¥¥ ¥¥
¥quad ¥cdots ¥¥ ¥¥
c_{NM}=a_{N1}b_{1M}+¥cdots+a_{NL}b_{LM}
¥end{array}¥right.
¥end{array}

〈6-8〉 (6.1.5)
¥mbox{When }
A=¥left(¥begin{array}{ccc}
1 & 2 & 3 ¥¥ ¥¥ 4 & 5 & 6
¥end{array}¥right),¥mbox{ then }
A^{T}=¥left(¥begin{array}{cc}
1 & 4 ¥¥ ¥¥ 2 & 5 ¥¥ ¥¥ 3 & 6
¥end{array}¥right)

〈6-9〉 (6.1.6)
(AB)^T=B^TA^T

〈6-10〉 (6.1.7)
E=¥left(¥begin{array}{cccc}
1 & 0 & 0 & ¥cdots ¥¥ ¥¥
0 & 1 & 0 & ¥cdots ¥¥ ¥¥
0 & 0 & 1 & ¥cdots ¥¥ ¥¥
¥cdots & ¥cdots & ¥cdots & ¥cdots
¥end{array}¥right)

〈6-11〉 (6.1.8)
AE=EA=A
```

〈6-12〉 (6.1.9)
```
O=¥left(¥begin{array}{ccc}
0 & 0 & ¥cdots ¥¥ ¥¥
0 & 0 & ¥cdots ¥¥ ¥¥
¥cdots & ¥cdots & ¥cdots
¥end{array}¥right)
```

〈6-13〉
```
¥frac{1}{a}=a^{-1}a=1
```

〈6-14〉
```
AB=BA=E
```

〈6-15〉 (6.1.10)
```
AA^{-1}=A^{-1}A=E
```

〈6-16〉
```
A=¥left(¥begin{array}{cc}
a_{11} & a_{12} ¥¥ ¥¥
a_{21} & a_{22}
¥end{array}¥right)
```

〈6-17〉
```
A^{-1}=¥frac{1}{¥mbox{det}(A)}¥left(¥begin{array}{cc
}
a_{22} & -a_{21} ¥¥ ¥¥
-a_{12} & a_{11}
¥end{array}¥right)
```

〈6-18〉
```
¥mbox{det}(A)=a_{11}a_{22}-a_{12}a_{21}
```

〈6-19〉 (6.1.11)
```
M=¥left(¥begin{array}{cccc}
p_{11} & p_{12} & ¥cdots & p_{1m} ¥¥ ¥¥
p_{21} & p_{22} & ¥cdots & p_{2m} ¥¥ ¥¥
¥cdots & ¥cdots & ¥cdots & ¥cdots ¥¥ ¥¥
p_{m1} & p_{m2} & ¥cdots & p_{mm}
¥end{array}¥right)
```

<6-20>　(6.1.12)
¥mbox{det}(M)=¥left|¥begin{array}{cccc}
p_{11} & p_{12} & ¥cdots & p_{1m} ¥¥ ¥¥
p_{21} & p_{22} & ¥cdots & p_{2m} ¥¥ ¥¥
¥cdots & ¥cdots & ¥cdots & ¥cdots ¥¥ ¥¥
p_{m1} & p_{m2} & ¥cdots & p_{mm}
¥end{array}¥right|

<6-21>　(6.1.13)
¥mbox{det}(M_2)=¥left|¥begin{array}{cc}
p_{11} & p_{12} ¥¥ ¥¥
p_{21} & p_{22}
¥end{array}¥right|=p_{11}p_{22}-p_{12}p_{21}

<6-22>　(6.1.14)
¥begin{array}{rl}
¥mbox{det}(M_3)&=¥left|¥begin{array}{ccc}
p_{11} & p_{12} & p_{13} ¥¥ ¥¥
p_{21} & p_{22} & p_{23} ¥¥ ¥¥
p_{31} & p_{32} & p_{33}
¥end{array}¥right| ¥¥ ¥¥
&=p_{11}p_{22}p_{33}+p_{12}p_{23}p_{31}+p_{13}p_{21}
p_{32}
¥¥ ¥¥
&-p_{13}p_{22}p_{31}-p_{12}p_{21}p_{33}-p_{11}p_{23}
p_{32}
¥end{array}

<6-23>　(6.1.15)
A^*=(¥overline{A})^T=¥overline{(A^T)}

<6-24>
A=¥left(¥begin{array}{cc}
¥cos¥theta & ¥sin¥theta ¥¥ ¥¥
-¥sin¥theta & ¥cos¥theta
¥end{array}¥right)

<6-25>
A^*=¥left(¥begin{array}{cc}

¥cos¥theta & -¥sin¥theta ¥¥ ¥¥
¥sin¥theta & ¥cos¥theta
¥end{array}¥right)

〈6-26〉
H=¥left(¥begin{array}{cc}
¥cos¥theta & i¥sin¥theta ¥¥ ¥¥
-i¥sin¥theta & ¥cos¥theta
¥end{array}¥right)

〈6-27〉
H^*=¥left(¥begin{array}{cc}
¥cos¥theta & i¥sin¥theta ¥¥ ¥¥
-i¥sin¥theta & ¥cos¥theta
¥end{array}¥right)

〈6-28〉
B=¥left(¥begin{array}{cc}
0 & 2 ¥¥ ¥¥
1 & 0
¥end{array}¥right)

〈6-29〉
¥left¥{¥begin{array}{l}
AB=¥left(¥begin{array}{cc}
¥sin¥theta & 2¥cos¥theta ¥¥ ¥¥
¥cos¥theta & -2¥sin¥theta
¥end{array}¥right) ¥¥ ¥¥
BA=¥left(¥begin{array}{cc}
-2¥sin¥theta & ¥cos¥theta ¥¥ ¥¥
¥cos¥theta & ¥sin¥theta
¥end{array}¥right)
¥end{array}¥right.

〈6-30〉
¥begin{array}{l}
A^T=¥left(¥begin{array}{cc}
¥cos¥theta & -¥sin¥theta ¥¥ ¥¥
¥sin¥theta & ¥cos¥theta
¥end{array}¥right) ¥¥ ¥¥

B^T=¥left(¥begin{array}{cc}
0 & 1 ¥¥ ¥¥
2 & 0
¥end{array}¥right)
¥end{array}¥right.

<6-31>
¥left¥{¥begin{array}{l}
(AB)^T=¥left(¥begin{array}{cc}
¥sin¥theta & ¥cos¥theta ¥¥ ¥¥
2¥cos¥theta & -2¥sin¥theta
¥end{array}¥right) ¥¥ ¥¥
B^TA^T=¥left(¥begin{array}{cc}
¥sin¥theta & ¥cos¥theta ¥¥ ¥¥
2¥cos¥theta & -2¥sin¥theta
¥end{array}¥right)
¥end{array}¥right.

<6-32>
(AB)^T=B^TA^T

<6-33>　(6.1.16)
¥begin{array}{l}
¥left¥{¥begin{array}{l}
a_{11}x+a_{12}y=b_1 ¥¥ ¥¥
a_{21}x+a_{22}y=b_2
¥end{array}¥right., ¥¥ ¥¥
¥mbox{with }¥quad
a_{11}a_{22}-a_{12}a_{21}¥ne0
¥quad¥mbox{assumed.}
¥end{array}

<6-34>　(6.1.17)
¥begin{array}{l}
¥displaystyle{x=
¥frac{a_{22}b_{1}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{2
1}}}
¥¥ ¥¥
¥displaystyle{y=
¥frac{a_{11}b_{2}-a_{21}b_1}{a_{11}a_{22}-a_{12}a_{2

```
1}}}
¥end{array}

<6-35>  (6.1.18)
¥begin{array}{l}
A=¥left(¥begin{array}{cc}
a_{11} & a_{12} ¥¥ ¥¥ a_{21} & a_{22}
¥end{array}¥right) ¥¥ ¥¥
X=¥left(¥begin{array}{c}
x ¥¥ ¥¥ y
¥end{array}¥right) ¥¥ ¥¥
B=¥left(¥begin{array}{c}
b_1 ¥¥ ¥¥ b_2
¥end{array}¥right)
¥end{array}

<6-36>  (6.1.19)
AX=B

<6-37>  (6.1.20)
X=A^{-1}B

<6-38>  (6.1.21)
A^{-1}=¥left[¥mbox{det}(A)¥right]^{-1}
¥left(¥begin{array}{cc}
a_{22} & -a_{12} ¥¥ ¥¥ -a_{21} & a_{11}
¥end{array}¥right)

<6-39>
¥mbox{det}(A)=a_{11}a_{22}-a_{12}a_{21}

<6-40>  (6.1.22)
X=¥frac{1}{a_{11}a_{22}-a_{12}a_{21}}
¥left(¥begin{array}{c}
a_{22}b_1-a_{12}b_2 ¥¥ ¥¥ -a_{21}b_1+a_{11}b_2
¥end{array}¥right)

<6-41>  (6.1.23)
¥begin{array}{l}
¥displaystyle{x=
```

¥frac{a_{22}b_{1}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{2
1}}}
¥¥ ¥¥
¥displaystyle{y=
¥frac{a_{11}b_{2}-a_{21}b_1}{a_{11}a_{22}-a_{12}a_{2
1}}}
¥end{array}

〈6-42〉 (6.1.24)
¥left¥{¥begin{array}{l}
a_{11}x_1+a_{12}x_2+¥cdots+a_{1N}x_N=b_1 ¥¥ ¥¥
a_{21}x_1+a_{22}x_2+¥cdots+a_{2N}x_N=b_2 ¥¥ ¥¥
¥cdots ¥¥ ¥¥
a_{N1}x_1+a_{N2}x_2+¥cdots+a_{NN}x_N=b_N
¥end{array}¥right.

〈6-43〉 (6.1.25)
X=A^{-1}B

〈6-44〉 (6.1.26)
¥begin{array}{l}
X=¥left(¥begin{array}{c}
x_1 ¥¥ ¥¥ x_2 ¥¥ ¥¥ ¥cdots ¥¥ ¥¥ x_N
¥end{array}¥right) ¥¥ ¥¥
A=¥left(¥begin{array}{cccc}
a_{11} & a_{12} & ¥cdots & a_{1N} ¥¥ ¥¥
a_{21} & a_{22} & ¥cdots & a_{2N} ¥¥ ¥¥
¥cdots & ¥cdots & ¥cdots & ¥cdots ¥¥ ¥¥
a_{N1} & a_{N2} & ¥cdots & a_{NN}
¥end{array}¥right),¥quad B=¥left(¥begin{array}{c}
b_1 ¥¥ ¥¥ b_2 ¥¥ ¥¥ ¥cdots ¥¥ ¥¥ b_N
¥end{array}¥right)
¥end{array}

〈6-45〉 (6.2.1)
¥left(¥begin{array}{c}
2 ¥¥ ¥¥ 3
¥end{array}¥right)=¥vec{a}

〈6-46〉 (6.2.2)

¥mbox{unit vector }(¥vec{i},¥vec{j}):¥quad
¥vec{i}=¥left(¥begin{array}{c}
1 ¥¥ ¥¥ 0
¥end{array}¥right),¥quad¥vec{j}=¥left(¥begin{array}{
c}
0 ¥¥ ¥¥ 1
¥end{array}¥right)

〈6-47〉　(6.2.3)
¥mbox{unit vector }(¥vec{e}_r,¥vec{e}_¥theta):¥quad
¥vec{e}_r=¥displaystyle{¥frac{1}{¥sqrt{2}}}
¥left(¥begin{array}{c}
1 ¥¥ ¥¥ i
¥end{array}¥right),¥quad
¥vec{e}_¥theta=
¥displaystyle{¥frac{1}{¥sqrt{2}}}
¥left(¥begin{array}{c}
1 ¥¥ ¥¥ -i
¥end{array}¥right)

〈6-48〉　(6.2.4)
¥begin{array}{rl}
¥vec{a} &=¥vec{i}2+¥vec{j}3 ¥¥ ¥¥
&=¥displaystyle{
¥vec{e}_r¥left(¥frac{2-3i}{¥sqrt{2}}¥right)
+¥vec{e}_¥theta¥left(¥frac{2+3i}{¥sqrt{2}}}¥right)
¥end{array}

〈6-49〉　(6.2.5)
¥vec{a}=¥vec{i}a_1+¥vec{j}a_2

〈6-50〉　(6.2.6)
¥left¥{¥begin{array}{l}
a_1=a¥cos¥theta ¥¥ ¥¥
a_2=a¥sin¥theta
¥end{array}¥right.

〈6-51〉　(6.2.7)
¥left¥{¥begin{array}{l}
¥displaystyle{a=¥sqrt{a_1^2+a_2^2}} ¥¥ ¥¥

¥displaystyle{¥tan¥theta=¥frac{a_2}{a_1}}¥quad
¥mbox{or}¥quad
¥displaystyle{¥theta
=¥tan^{-1}¥left(¥frac{a_2}{a_1}¥right)}
¥end{array}¥right.

〈6-52〉 (6.2.8)
¥vec{a}=¥left(¥begin{array}{c}
a_1 ¥¥ ¥¥ a_2
¥end{array}¥right),¥quad ¥vec{b}
=¥left(¥begin{array}{c}
b_1 ¥¥ ¥¥ b_2
¥end{array}¥right)

〈6-53〉 (6.2.9)
¥left¥{¥begin{array}{l}
¥vec{a}=¥vec{i}a_1+¥vec{j}a_2 ¥¥ ¥¥
¥vec{b}=¥vec{i}b_1+¥vec{j}b_2
¥end{array}¥right.

〈6-54〉 (6.2.10)
¥vec{a}¥pm¥vec{b}=¥vec{i}(a_1¥pm b_1)+
¥vec{j}(a_2¥pm b_2)

〈6-55〉 (6.2.11)
¥vec{a}=¥left(¥begin{array}{c}
1 ¥¥ ¥¥ 2
¥end{array}¥right),¥quad ¥vec{b}
=¥left(¥begin{array}{c}
2 ¥¥ ¥¥ 1
¥end{array}¥right)

〈6-56〉 (6.2.12)
¥vec{c}=¥vec{a}+¥vec{b}=¥left(¥begin{array}{c}
3 ¥¥ ¥¥ 3
¥end{array}¥right)

〈6-57〉 (6.2.13)
¥vec{a}-¥vec{b}=¥vec{a}+(-¥vec{b})

〈6-58〉　(6.2.14)
¥vec{a}¥cdot¥vec{b}=a_1b_1+a_2b_2

〈6-59〉　(6.2.15)
¥left¥{¥begin{array}{l}
(¥vec{i}¥cdot¥vec{i})=(¥vec{j}¥cdot¥vec{j})=1
¥¥ ¥¥
(¥vec{i}¥cdot¥vec{j})=(¥vec{j}¥cdot¥vec{i})=0
¥end{array}¥right.

〈6-60〉
¥begin{array}{rl}
(¥vec{a}¥cdot¥vec{b})&=(¥vec{i}a_1
+¥vec{j}a_2)¥cdot(¥vec{i}b_1+¥vec{j}b_2) ¥¥ ¥¥
&=(¥vec{i}¥cdot¥vec{i})a_1b_1
+(¥vec{i}¥cdot¥vec{j})a_1b_2
+(¥vec{j}¥cdot¥vec{i})a_2b_1
+(¥vec{j}¥cdot¥vec{j})a_2b_2 ¥¥ ¥¥
&=a_1b_1+a_2b_2
¥end{array}

〈6-61〉　(6.2.16)
¥left¥{¥begin{array}{l}
(a_1=a¥cos¥theta_a,a_2=a¥sin¥theta_a) ¥¥ ¥¥
(b_1=b¥cos¥theta_b,b_2=b¥sin¥theta_b)
¥end{array}¥right.

〈6-62〉　(6.2.17)
¥begin{array}{rl}
(¥vec{a}¥cdot¥vec{b})
&=ab(¥cos¥theta_a¥cos¥theta_b
+¥sin¥theta_a¥sin¥theta_b ¥¥ ¥¥
&=ab¥cos(¥theta_a-¥theta_b) ¥¥ ¥¥
&=ab¥cos¥theta
¥end{array}

〈6-63〉　(6.2.18)
¥cos(¥theta_1¥pm¥theta_2)=
¥cos¥theta_1¥cos¥theta_2
¥mp¥sin¥theta_1¥sin¥theta_2

〈6-64〉　(6.2.19)
¥vec{a}¥cdot¥vec{b}=a(b¥cos¥theta)
=b(a¥cos¥theta)

〈6-65〉　(6.2.20)
a=¥sqrt{¥vec{a}¥cdot¥vec{a}}

〈6-66〉　(6.2.21)
¥vec{a}=¥vec{i}a_1+¥vec{j}a_2

〈6-67〉　(6.2.22)
¥left¥{¥begin{array}{l}
(¥vec{i}¥cdot¥vec{i})=(¥vec{j}¥cdot¥vec{j})
=1 ¥¥ ¥¥
(¥vec{i}¥cdot¥vec{j})=(¥vec{j}¥cdot¥vec{i})
=0
¥end{array}¥right.

〈6-68〉　(6.2.23)
¥left¥{¥begin{array}{l}
¥vec{a}=¥vec{i}a_1+¥vec{j}a_2 ¥¥ ¥¥
¥vec{b}=¥vec{i}b_1+¥vec{j}b_2
¥end{array}¥right.

〈6-69〉　(6.2.24)
¥begin{array}{rl}
¥vec{a}¥cdot¥vec{b}&=
(¥vec{i}a_1+¥vec{j}a_2)¥cdot
(¥vec{i}b_1+¥vec{j}b_2) ¥¥ ¥¥
&=a_1b_1+a_2b_2
¥end{array}¥right.

〈6-70〉　(6.2.25)
¥vec{r}=¥vec{i}x+¥vec{j}y

〈6-71〉　(6.2.26)
¥left¥{¥begin{array}{l}
x'=x¥cos¥theta-y¥sin¥theta ¥¥ ¥¥
y'=x¥sin¥theta+y¥cos¥theta

¥end{array}¥right.

〈6-72〉
¥mbox{(a)}¥quad¥left¥{¥begin{array}{l}
x=r¥cos¥theta_0 ¥¥ ¥¥
y=r¥sin¥theta_0
¥end{array}¥right.

〈6-73〉
¥mbox{(b)}¥quad¥left¥{¥begin{array}{rl}
x'&=r¥cos(¥theta_0+¥theta) ¥¥ ¥¥
&=r(¥cos¥theta_0¥cos¥theta-
¥sin¥theta_0¥sin¥theta) ¥¥ ¥¥
&=x¥cos¥theta-y¥sin¥theta ¥¥ ¥¥
y'&=r¥sin(¥theta_0+¥theta) ¥¥ ¥¥
&=r(¥sin¥theta_0¥cos¥theta-
¥cos¥theta_0¥sin¥theta) ¥¥ ¥¥
&=y¥cos¥theta+x¥sin¥theta
¥end{array}¥right.

〈6-74〉
¥mbox{(c)}¥quad¥left¥{¥begin{array}{l}
¥sin(a¥pm b)=¥sin a¥cos b¥pm¥cos a¥sin b ¥¥ ¥¥
¥cos(a¥pm b)=¥cos a¥cos b¥mp¥sin a¥sin b
¥end{array}¥right.

〈6-75〉 (6.2.27)
¥left(¥begin{array}{c}
x' ¥¥ ¥¥ y'
¥end{array}¥right)=¥left(¥begin{array}{cc}
¥cos¥theta & -¥sin¥theta ¥¥ ¥¥
¥sin¥theta & ¥cos¥theta
¥end{array}¥right)¥left(¥begin{array}{c}
x ¥¥ ¥¥ y
¥end{array}¥right)

〈6-76〉 (6.2.28)
R(¥theta)=¥left(¥begin{array}{cc}
¥cos¥theta & -¥sin¥theta ¥¥ ¥¥
¥sin¥theta & ¥cos¥theta

¥end{array}¥right)

<6-77>　(6.2.29)
¥left(¥begin{array}{c}
x ¥¥ ¥¥ y
¥end{array}¥right)¥rightarrow
¥left(¥begin{array}{c}
x' ¥¥ ¥¥ y'
¥end{array}¥right)=R(¥theta_1)
¥left(¥begin{array}{c}
x ¥¥ ¥¥ y
¥end{array}¥right)

<6-78>　(6.2.30)
¥left(¥begin{array}{c}
x' ¥¥ ¥¥ y'
¥end{array}¥right)¥rightarrow
¥left(¥begin{array}{c}
x'' ¥¥ ¥¥ y''
¥end{array}¥right)=R(¥theta_2)
¥left(¥begin{array}{c}
x' ¥¥ ¥¥ y'
¥end{array}¥right)=R(¥theta_2)R(¥theta_1)
¥left(¥begin{array}{c}
x ¥¥ ¥¥ y
¥end{array}¥right)

<6-79>　(6.2.31)
R(¥theta_2)R(¥theta_1)=
¥left(¥begin{array}{cc}
¥cos¥theta_2 & -¥sin¥theta_2 ¥¥ ¥¥
¥sin¥theta_2 & ¥cos¥theta_2
¥end{array}¥right)¥left(¥begin{array}{cc}
¥cos¥theta_1 & -¥sin¥theta_1 ¥¥ ¥¥
¥sin¥theta_1 & ¥cos¥theta_1
¥end{array}¥right)

<6-80>　(6.2.32)
¥begin{array}{l}
R(¥theta_2)R(¥theta_1) ¥¥ ¥¥

=¥left(¥begin{array}{cc}
(¥cos¥theta_1¥cos¥theta_2-¥sin¥theta_1¥sin¥theta_2)
&
(-¥sin¥theta_1¥cos¥theta_2-
¥cos¥theta_1¥sin¥theta_2) ¥¥ ¥¥
(¥cos¥theta_1¥sin¥theta_2+
¥sin¥theta_1¥cos¥theta_2) &
(-¥sin¥theta_1¥sin¥theta_2+
¥cos¥theta_1¥cos¥theta_2)
¥end{array}¥right)

<6-81> (6.2.33)
¥begin{array}{l}
R(¥theta_2)R(¥theta_1) ¥¥ ¥¥
=¥left(¥begin{array}{cc}
¥cos(¥theta_1+¥theta_2) &
-¥sin(¥theta_1+¥theta_2) ¥¥ ¥¥
¥sin(¥theta_1+¥theta_2) &
¥cos(¥theta_1+¥theta_2)
¥end{array}¥right)

<6-82> (6.2.34)
¥vec{r}(t)=¥vec{i}x(t)+¥vec{j}y(t)

<6-83> (6.2.35)
¥vec{v}(t)=¥vec{i}¥frac{dx(t)}{dt}
+¥vec{j}¥frac{dy(t)}{dt}

<6-84> (6.2.36)
¥vec{v}(t)=¥frac{d¥vec{r}(t)}{dt}

<6-85> (6.2.37)
¥vec{v}(t)=¥vec{i}v_x(t)+¥vec{j}v_y(t)

<6-86> (6.2.38)
v(t)=¥sqrt{v_x(t)^2+v_y(t)^2}

<6-87> (6.2.39)
¥vec{a}(t)=¥frac{d¥vec{v}(t)}{dt}=
¥frac{d^2¥vec{r}(t)}{dt^2}

〈6-88〉　(6.2.40)
¥vec{a}(t)=¥vec{i}a_x(t)+¥vec{j}a_y(t)

〈6-89〉
a(t)=¥sqrt{a_x(t)^2+a_y(t)^2}

〈6-90〉
v(t)=¥sqrt{v_x(t)^2+v_y(t)^2}

〈6-91〉
¥vec{a}(t)=¥frac{d¥vec{v}(t)}{dt}

〈6-92〉
a(t)¥ne¥frac{dv(t)}{dt}

〈6-93〉　(6.2.41)
¥vec{r}=¥vec{i}x+¥vec{j}y

〈6-94〉　(6.2.42)
¥begin{array}{rl}
¥displaystyle{¥vec{i}¥frac{¥partial A}{¥partial x}+
¥vec{j}}¥frac{¥partial A}{¥partial y}
&=¥displaystyle{¥left(
¥vec{i}¥frac{¥partial}{¥partial x}
+¥vec{j}¥frac{¥partial}{¥partial y}¥right)
A(¥vec{r},t)}¥¥ ¥¥
&¥displaystyle{¥equiv¥nabla A¥quad¥mbox{or}
¥quad ¥mbox{grad}A}
¥end{array}

〈6-95〉　(6.2.43)
¥nabla=¥vec{i}¥frac{¥partial}{¥partial x}+
¥vec{j}¥frac{¥partial}{¥partial y}

〈6-96〉　(6.2.44)
¥left¥{¥begin{array}{l}
T_x(¥vec{r})=T(¥vec{r})¥cos¥theta ¥¥ ¥¥
T_y(¥vec{r})=T(¥vec{r})¥sin¥theta
¥end{array}¥right.

〈6-97〉　(6.2.45)
¥vec{r}=¥vec{i}T_x(¥vec{r})+¥vec{j}T_y(¥vec{r})

〈6-98〉
¥left(¥begin{array}{cc}
¥displaystyle{¥frac{¥partial T_x}{¥partial x}}
& ¥displaystyle{¥frac{¥partial T_x}{¥partial y}} ¥¥
¥¥
¥displaystyle{¥frac{¥partial T_y}{¥partial x}}
& ¥displaystyle{¥frac{¥partial T_y}{¥partial y}}
¥end{array}¥right)

〈6-99〉　(6.2.46)
¥begin{array}{rl}
¥mbox{div}¥vec{T}(¥vec{r})
&=¥displaystyle{
¥frac{¥partial T_x(¥vec{r})}{¥partial x}+
¥frac{¥partial T_y(¥vec{r})}{¥partial y}}¥¥ ¥¥
&=¥displaystyle{
¥frac{¥partial}{¥partial x}T_x(¥vec{r})+
¥frac{¥partial}{¥partial y}T_y(¥vec{r})} ¥¥ ¥¥
&=¥displaystyle{
¥left(¥vec{i}¥frac{¥partial}{¥partial x}+
¥vec{j}¥frac{¥partial}{¥partial y}¥right)}
¥cdot¥left(¥vec{i}T_x+¥vec{j}T_y¥right)
¥end{array}

〈6-100〉　(6.2.47)
¥mbox{div}¥vec{T}(¥vec{r})
=¥nabla¥cdot¥vec{T}

〈6-101〉　(6.3.1)
¥left¥{¥begin{array}{l}
¥vec{i}¥cdot¥vec{i}=¥vec{j}¥cdot¥vec{j}
=¥vec{k}¥cdot¥vec{k}=
1 ¥¥ ¥¥
¥vec{i}¥cdot¥vec{j}=¥vec{j}¥cdot¥vec{k}
=¥vec{k}¥cdot¥vec{i}=0
¥end{array}¥right.

〈6-102〉　(6.3.2)
¥left¥{¥begin{array}{l}
¥vec{i}¥times¥vec{i}=¥vec{j}¥times¥vec{j}
=¥vec{k}¥times¥vec{k}=
0 ¥¥ ¥¥
¥vec{i}¥times¥vec{j}=¥vec{k},
¥;¥vec{j}¥times¥vec{k}=¥vec{i},¥;
¥vec{k}¥times¥vec{i}=¥vec{j}
¥end{array}¥right.

〈6-103〉
¥vec{b}¥times¥vec{a}=-¥vec{a}¥times¥vec{b}

〈6-104〉　(6.3.3)
¥vec{T}=¥vec{i}T_1+¥vec{j}T_2+¥vec{k}T_3

〈6-105〉　(6.3.4)
¥left¥{¥begin{array}{rl}
¥vec{a}¥cdot¥vec{b}&=(¥vec{i}a_1
+¥vec{j}a_2+¥vec{k}a_3)¥cdot
(¥vec{i}b_1+¥vec{j}b_2+¥vec{k}b_3) ¥¥ ¥¥
&=a_1b_1+a_2b_2+a_3b_3
¥end{array}¥right.

〈6-106〉　(6.3.5)
¥vec{a}¥cdot¥vec{b}=ab¥cos¥theta

〈6-107〉　(6.3.6)
¥left¥{¥begin{array}{rl}
¥vec{a}¥times¥vec{b}&=(¥vec{i}a_1
+¥vec{j}a_2+¥vec{k}a_3)¥times
(¥vec{i}b_1+¥vec{j}b_2+¥vec{k}b_3) ¥¥ ¥¥
&=¥vec{i}(a_2b_3-a_3b_2)
+¥vec{j}(a_3b_1-a_1b_3)+
¥vec{k}(a_1b_2-a_2b_1)
¥end{array}¥right.

〈6-108〉　(6.3.7)
¥vec{r}=¥vec{i}x+¥vec{j}y+¥vec{k}z

〈6-109〉　(6.3.8)
¥nabla A(¥vec{r})
=¥vec{i}¥frac{¥partial A}{¥partial x}
+¥vec{j}¥frac{¥partial A}{¥partial y}}
+¥vec{k}¥frac{¥partial A}{¥partial z}

〈6-110〉　(6.3.9)
¥vec{T}(¥vec{r})=¥vec{i}T_1(¥vec{r})
+¥vec{j}T_2(¥vec{r})+¥vec{k}T_3(¥vec{r})

〈6-111〉　(6.3.10)
¥mbox{div}¥vec{T}(¥vec{r})=
¥frac{¥partial T_1}{¥partial x}+
¥frac{¥partial T_2}{¥partial y}}+
¥frac{¥partial T_3}{¥partial z}

〈6-112〉　(6.3.11)
¥nabla=¥vec{i}¥frac{¥partial}{¥partial x}+
¥vec{j}¥frac{¥partial}{¥partial y}+
¥vec{k}¥frac{¥partial}{¥partial z}

〈6-113〉　(6.3.12)
¥mbox{div}¥vec{T}(¥vec{r})
=¥nabla¥cdot¥vec{T}(¥vec{r})

〈6-114〉
¥begin{array}{rl}
¥mbox{rot}¥vec{T}(¥vec{r})&=
¥nabla¥times¥vec{T}(¥vec{r}) ¥¥ ¥¥
&=¥left(¥vec{i}¥frac{¥partial}{¥partial x}+
¥vec{j}¥frac{¥partial}{¥partial y}
+¥vec{k}¥frac{¥partial}{¥partial z}¥right) ¥¥ ¥¥
&¥quad¥times¥left(¥vec{i}T_x}+¥vec{j}T_y
+¥vec{k}T_z¥right) ¥¥ ¥¥
&=(¥vec{i}¥times
¥vec{i})¥frac{¥partial T_x}{¥partial x}
+(¥vec{i}¥times
¥vec{j})¥frac{¥partial T_y}{¥partial x}
+(¥vec{i}¥times

```
¥vec{k})¥frac{¥partial T_z}{¥partial x} ¥¥ ¥¥
&=(¥vec{j}¥times¥vec{i})
¥frac{¥partial T_x}{¥partial y}+
(¥vec{j}¥times¥vec{j})
¥frac{¥partial T_y}{¥partial y}
+(¥vec{j}¥times¥vec{k})
¥frac{¥partial T_z}{¥partial y} ¥¥ ¥¥
&=(¥vec{k}¥times¥vec{i})
¥frac{¥partial T_x}{¥partial z}+
(¥vec{k}¥times¥vec{j})
¥frac{¥partial T_y}{¥partial z}
+(¥vec{k}¥times¥vec{k})
¥frac{¥partial T_z}{¥partial z}
¥end{array}
```

〈6-115〉　(6.3.13)

```
¥mbox{rot}¥vec{T}(¥vec{r})=¥vec{i}¥left[
¥frac{¥partial T_z}{¥partial y}-
¥frac{¥partial T_y}{¥partial z}¥right]
+¥vec{j}¥left[¥frac{¥partial T_x}{¥partial z}
-¥frac{¥partial T_z}{¥partial x}¥right]
+¥vec{k}¥left[¥frac{¥partial T_y}{¥partial x}
-¥frac{¥partial T_x}{¥partial y}¥right]
```

〈6-116〉

```
¥mbox{grad}f(¥vec{r})=¥nabla f(¥vec{r})
=¥vec{i}¥frac{¥partial f}{¥partial x}
+¥vec{j}¥frac{¥partial f}{¥partial y}
+¥vec{k}¥frac{¥partial f}{¥partial z}
```

〈6-117〉

```
¥mbox{div}¥vec{a}(¥vec{r})=¥nabla¥cdot¥vec{a}(¥vec{r
})
=¥frac{¥partial a_x}{¥partial x}
+¥frac{¥partial a_y}{¥partial y}
+¥frac{¥partial a_z}{¥partial z}
```

〈6-118〉

```
¥begin{array}{rl}
¥mbox{rot}¥vec{a}(¥vec{r})&=
```

¥nabla¥times¥vec{a}(¥vec{r}) ¥¥ ¥¥
&=¥displaystyle{
¥vec{i}¥left(¥frac{¥partial a_z}{¥partial y}-
¥frac{¥partial a_y}{¥partial z}¥right)
+¥vec{j}¥left(¥frac{¥partial a_x}{¥partial z}-
¥frac{¥partial a_z}{¥partial x}¥right)
+¥vec{k}¥left(¥frac{¥partial a_y}{¥partial x}-
¥frac{¥partial a_x}{¥partial y}¥right)}
¥end{array}

〈6-119〉 (6.3.14)
¥frac{d}{dt}(f¥vec{a})
=¥frac{df}{dt}¥vec{a}+f¥frac{d¥vec{a}}{dt}

〈6-120〉 (6.3.15)
¥frac{d(¥vec{a}¥cdot¥vec{b})}{dt}
=¥frac{d¥vec{a}}{dt}¥cdot¥vec{b}+
¥vec{a}¥cdot¥frac{d¥vec{b}}{dt}

〈6-121〉 (6.3.16)
¥frac{d[¥vec{a}¥times¥vec{b}]}{dt}=
¥frac{d¥vec{a}}{dt}¥times¥vec{b}
+¥vec{a}¥times¥frac{d¥vec{b}}{dt}

〈6-122〉
¥vec{a}¥cdot¥vec{a}

〈6-123〉 (6.3.17)
¥vec{a}¥cdot¥vec{a}=¥mbox{constant}
¥quad¥Rightarrow¥quad
¥vec{a}¥cdot¥frac{¥partial¥vec{a}}{¥partial t}
=0

〈6-124〉 (6.3.18)
¥mbox{rot}¥vec{r}=¥nabla¥times¥vec{r}=0

〈6-125〉
¥vec{a}(¥vec{r},t)=¥nabla f(¥vec{r},t)

〈6-126〉 (6.3.19)

¥mbox{rot}¥vec{a}=¥nabla¥times¥vec{a}=0

＜6-127＞　(6.3.20)
¥begin{array}{rl}
¥nabla¥cdot(¥nabla f)
&=¥displaystyle{¥frac{¥partial^2f}{¥partial x^2}
+¥frac{¥partial^2f}{¥partial y^2}+
¥frac{¥partial^2f}{¥partial z^2}} ¥¥ ¥¥
&=¥displaystyle{
¥left(¥frac{¥partial^2}{¥partial x^2}
+¥frac{¥partial^2}{¥partial y^2}
+¥frac{¥partial^2}{¥partial z^2}¥right)f} ¥¥ ¥¥
&¥displaystyle{¥equiv¥nabla f}
¥end{array}

＜6-128＞　(6.3.21)
¥Delta¥left(¥frac{1}{r}¥right)=0

＜6-129＞　(6.3.22)
¥nabla(fg)=f(¥nabla g)+(¥nabla f)g

＜6-130＞　(6.3.23)
¥nabla¥cdot(f¥vec{a})=f(¥nabla g)+(¥nabla f)g

＜6-131＞　(6.3.24)
¥nabla¥times(f¥vec{a})=(¥nabla f)¥times¥vec{a}
+f(¥nabla¥times¥vec{a})

＜6-132＞　(6.3.25)
¥nabla¥cdot(¥vec{a}¥times¥vec{b})=
(¥nabla¥tiems¥vec{a})¥cdot¥vec{b}-
¥vec{a}¥cdot(¥nabla¥times¥vec{b})

＜6-133＞　(6.3.26)
¥nabla¥times[¥vec{a}¥times¥vec{b}]
=(¥vec{b}¥cdot¥nabla)¥vec{a}
-¥vec{b}(¥nabla¥cdot¥vec{a})
-(¥vec{a}¥cdot¥nabla)¥vec{b}
+¥vec{a}(¥nabla¥cdot¥vec{b})

```
〈6-134〉　(6.3.27)
¥nabla(¥vec{a}¥cdot¥vec{b})
=(¥vec{b}¥cdot¥nabla)¥vec{a}
+(¥vec{a}¥cdot¥nabla)¥vec{b}+
¥vec{b}¥times(¥nabla¥times¥vec{a})
+¥vec{a}¥times(¥nabla¥times¥vec{b})

〈6-135〉　(6.3.28)
¥nabla¥times(¥nabla f)
=¥mbox{rot}(¥mbox{grad}f)=0

〈6-136〉　(6.3.29)
¥nabla¥cdot(¥nabla¥times¥vec{a})
=¥mbox{div}(¥mbox{rot}¥vec{a})=0

〈6-137〉　(6.3.30)
¥begin{array}{l}
¥displaystyle{¥nabla¥times[¥nabla¥times¥vec{a}]
=¥nabla(¥nabla¥cdot¥vec{a})-¥nabla^2¥vec{a}} ¥¥ ¥¥
¥mbox{where}¥quad¥displaystyle{¥nabla^2¥vec{a}
=¥frac{¥partial^2¥vec{a}}{¥partial x^2}
+¥frac{¥partial^2¥vec{a}}{¥partial y^2}
+¥frac{¥partial^2¥vec{a}}{¥partial z^2}}
¥end{array}

〈7-1〉　(7.1.1)
F(x)=¥int f(x)dx

〈7-2〉　(7.1.2)
f(x)=f(g(y))

〈7-3〉　(7.1.3)
dx=¥frac{dx}{dy}dy=¥frac{dg(y)}{dy}dy

〈7-4〉　(7.1.4)
F(x)=¥int¥left[f(g(y))¥frac{dg(y)}{dy}¥right]dy

〈7-5〉　(7.1.5)
dx=¥frac{dx}{dy}dy
```

〈7-6〉

```
¥begin{array}{rl}
I&=¥displaystyle{¥int x^3dx} ¥¥ ¥¥
&=¥displaystyle{¥int(¥pm¥sqrt{y})^3
¥left(¥pm¥frac{1}{2¥sqrt{y}}¥right)dy} ¥¥ ¥¥
&=¥displaystyle{¥frac{1}{2}¥int ydy} ¥¥ ¥¥
&=¥displaystyle{¥frac{1}{2}¥frac{y^2}{2}} ¥¥ ¥¥
&=¥displaystyle{¥frac{y^2}{4}} ¥¥ ¥¥
&=¥displaystyle{¥frac{x^4}{4}}
¥end{array}
```

〈7-7〉 (7.1.6)

```
¥left¥{¥begin{array}{rl}
¥sin 2x&=2¥sin x¥cos x ¥¥ ¥¥
¥cos 2x&=¥cos^2x-¥sin^2x ¥¥ ¥¥
&=2¥cos^2x-1 ¥¥ ¥¥
&=1-2¥sin^2x
¥end{array}¥right.
```

〈7-8〉 (7.1.7)

```
¥sin^2x=¥displaystyle{¥frac{1}{2}(1-¥cos2x)}
```

〈7-9〉

```
¥begin{array}{rl}
I&=¥displaystyle{¥int¥sin^2xdx} ¥¥ ¥¥
&=¥displaystyle{
¥frac{1}{2}¥int¥left[1-¥cos(2x)¥right]dx} ¥¥ ¥¥
&=¥displaystyle{
¥frac{1}{2}¥left[¥int1dx-¥int¥cos(2x)dx¥right]}
¥end{array}
```

〈7-10〉

```
¥begin{array}{rl}
¥displaystyle{¥int¥cos(2x)dx}&=
¥displaystyle{¥int¥cos y¥frac{dx}{dy}dy} ¥¥ ¥¥
&=¥displaystyle{¥frac{1}{2}¥int¥cos ydy} ¥¥ ¥¥
&=¥displaystyle{¥frac{1}{2}¥sin y}
¥end{array}
```

〈7-11〉

I=¥frac{x}{2}-¥frac{1}{4}¥sin 2x

〈7-12〉　（7.1.8）
I=¥int¥sqrt{a^2+x^2}dx

〈7-13〉
1+¥tan^2y=¥frac{1}{¥cos^2y}

〈7-14〉
¥begin{array}{rl}
¥displaystyle{¥sqrt{a^2+x^2}} &=
a¥displaystyle{¥sqrt{1+¥tan^2y}} ¥¥ ¥¥
&=¥displaystyle{¥frac{a}{¥cos y}}
¥end{array}

〈7-15〉
¥begin{array}{rl}
¥displaystyle{¥frac{dx}{dy}}&=
a¥displaystyle{¥frac{d¥tan y}{dy}} ¥¥ ¥¥
&=¥displaystyle{¥frac{a}{¥cos^2y}}
¥end{array}

〈7-16〉
I=a^2¥int¥frac{1}{¥cos^3y}dy

〈7-17〉
¥frac{1}{¥cos^3y}=
¥displaystyle{¥frac{1}{¥left(¥sqrt{1-z^2}¥right)^3}}

〈7-18〉
¥begin{array}{rl}
¥displaystyle{¥frac{dy}{dz}}&=
¥displaystyle{
¥frac{1}{¥displaystyle{¥frac{dz}{dy}}}} ¥¥ ¥¥
&=¥displaystyle{¥frac{1}{¥cos y}} ¥¥ ¥¥
&=¥displaystyle{¥frac{1}{¥sqrt{1-z^2}}}
¥end{array}

〈7-19〉
¥begin{array}{rl}

I&=¥displaystyle{
a^2¥int¥frac{1}{(¥sqrt{1-z^2})^3}
¥frac{1}{¥sqrt{1-z^2}}dz} ¥¥ ¥¥
&=¥displaystyle{
a^2¥int¥frac{1}{(1-z^2)^2}dz}
¥end{array}

〈7-20〉
¥begin{array}{rl}
¥displaystyle{¥frac{1}{(1-z^2)^2}}
&=¥displaystyle{¥left[¥frac{1}{2}
¥left(¥frac{1}{1+z}
+¥frac{1}{1-z}¥right)¥right]^2} ¥¥ ¥¥
&=¥displaystyle{
¥frac{1}{4}¥left[¥frac{1}{(1+z)^2}+
¥frac{1}{(1-z)^2}+¥frac{2}{1-z^2}¥right]}
¥end{array}

〈7-21〉
¥begin{array}{rl}
¥displaystyle{¥int¥frac{1}{(1+z^2)}dz
+¥int¥frac{1}{(1-z^2)}dz}
&=¥displaystyle{-¥frac{1}{1+z}
+¥frac{1}{1-z}¥right]} ¥¥ ¥¥
&=¥displaystyle{¥frac{2z}{1-z^2}}
¥end{array}

〈7-22〉
¥begin{array}{rl}
¥displaystyle{¥frac{2}{1-z^2}}
&=¥displaystyle{¥frac{1}{1-z}
+¥frac{1}{1+z}} ¥¥ ¥¥
&=¥displaystyle{¥frac{1}{z+1}
-¥frac{1}{z-1}}
¥end{array}

〈7-23〉
¥begin{array}{rl}
¥displaystyle{¥int¥frac{2}{1-z^2}dz}
&=¥displaystyle{¥int¥frac{1}{z+1}dz

```
-¥int¥frac{1}{z-1dz} ¥¥ ¥¥
&=¥displaystyle{¥ln|z+1|-¥ln|z-1|} ¥¥ ¥¥
&=¥displaystyle{¥ln¥left|
¥frac{z+1}{z-1}¥right|}
¥end{array}
```

〈7-24〉

```
I=¥frac{a^2}{4}¥frac{2z}{1-z^2}+
¥frac{a^2}{4}
¥ln¥left|¥frac{z+1}{z-1}¥right|
```

〈7-25〉

```
¥begin{array}{rl}
x^2&=a^2¥displaystyle{
¥frac{¥sin^2y}{¥cos^2y}} ¥¥ ¥¥
&=a^2¥displaystyle{¥frac{¥frac{z^2}{1-z^2}}
¥end{array}
```

〈7-26〉

```
z=¥frac{x}{¥sqrt{a^2+x^2}}
```

〈7-27〉

```
¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{2z}{1-z^2}=
¥frac{2x¥sqrt{a^2+x^2}}{a^2}} ¥¥ ¥¥
¥displaystyle{¥frac{z+1}{z-1}=
¥frac{¥left(¥sqrt{a^2+x^2}+x¥right)^2}{a^2}}
¥end{array}¥right.
```

〈7-28〉

```
¥begin{array}{rl}
¥displaystyle{¥int¥sqrt{a^2+x^2}dx}&=
¥displaystyle{¥frac{x}{2}¥sqrt{a^2+x^2}} ¥¥ ¥¥
&¥quad+¥displaystyle{
¥frac{a^2}{2}¥ln¥left|¥sqrt{a^2+x^2}
+x¥right|-¥frac{a^2}{2}¥ln a}
¥end{array}
```

〈7-29〉 (7.1.9)

```
¥frac{d¥{f(x)g(x)¥}}{dx}=¥frac{df(x)}{dx}g(x)
```

+f(x)¥frac{dg(x)}{dx}

〈7-30〉
f(x)g(x)=¥int¥frac{df(x)}{dx}g(x)dx
+¥int f(x)¥frac{dg(x)}{dx}dx

〈7-31〉
¥int f(x)¥frac{dg(x)}{dx}dx=f(x)g(x)
-¥int¥frac{df(x)}{dx}g(x)dx

〈7-32〉
I=¥int x¥cos xdx

〈7-33〉
¥begin{array}{rl}
¥displaystyle{¥int x¥cos xdx}&=
¥displaystyle{x¥sin x-¥int¥sin xdx} ¥¥ ¥¥
&=¥displaystyle{x¥sin x+¥cos x}
¥end{array}

〈7-34〉
I=¥int x¥ln xdx

〈7-35〉
¥begin{array}{rl}
¥displaystyle{¥int x¥ln xdx}&=
¥displaystyle{¥frac{x^2}{2}¥ln x-
¥int¥frac{1}{x}¥frac{x^2}{2}dx} ¥¥ ¥¥
&=¥displaystyle{¥frac{x^2}{2}¥ln x
-¥frac{1}{2}¥int xdx} ¥¥ ¥¥
&=¥displaystyle{¥frac{x^2}{2}¥ln x
-¥frac{x^2}{4}}
¥end{array}

〈7-36〉 (7.2.1)
¥int_b^a f(x)dx=F(a)-F(b)

〈7-37〉 (7.2.2)
I=¥int_C¥vec{A}(¥vec{r})¥cdot d¥vec{s}

〈7-38〉　(7.2.3)
¥vec{A}(¥vec{r})=
¥vec{i}A_x(x,y,z)+¥vec{j}A_y(x,y,z)
+¥vec{k}A_z(x,y,z)

〈7-39〉　(7.2.4)
d¥vec{s}=¥vec{i}dx+¥vec{j}dy+¥vec{k}dz

〈7-40〉
¥left¥{¥begin{array}{l}
dx=(¥vec{i}¥cdot d¥vec{s}) ¥¥ ¥¥
dy=(¥vec{j}¥cdot d¥vec{s}) ¥¥ ¥¥
dz=(¥vec{k}¥cdot d¥vec{s})
¥end{array}¥right.

〈7-41〉　(7.2.5)
I=¥int_CA_x(x,y,z)dx+¥int_CA_y(x,y,z)dy
+¥int_CA_z(x,y,z)dz

〈7-42〉　(7.2.6)
¥vec{e}_r=¥frac{¥vec{r}}{r}
=¥vec{i}¥frac{x}{r}+¥vec{j}¥frac{y}{r}

〈7-43〉
¥vec{A}(¥vec{r})=¥vec{e}_r¥sqrt{x}

〈7-44〉
¥left¥{¥begin{array}{l}
A_x=¥displaystyle{¥frac{x¥sqrt{x}}{r}
=¥frac{x}{¥sqrt{x+1}}} ¥¥ ¥¥
A_y=¥displaystyle{¥frac{y¥sqrt{x}}{r}
=¥frac{y}{¥sqrt{x+1}}
=¥frac{y}{¥sqrt{y^2+1}}}
¥end{array}¥right.

〈7-45〉
I=¥int_0^3¥frac{x}{¥sqrt{x+1}}dx
+¥int_0^{¥sqrt{3}}¥frac{y}{¥sqrt{y^2+1}}dy

〈7-46〉

¥left¥{¥begin{array}{l}
I_1=¥displaystyle{
¥int_0^3¥frac{x}{¥sqrt{x+1}}dx} ¥¥ ¥¥
I_2=¥displaystyle{
¥int_0^{¥sqrt{3}}¥frac{y}{¥sqrt{y^2+1}}dy}
¥end{array}¥right.

〈7-47〉
dy=¥frac{dy}{dx}dx=¥frac{1}{2¥sqrt{x}}dx

〈7-48〉
I_2=¥int_0^3¥frac{¥sqrt{x}}{¥sqrt{x+1}}
¥frac{1}{2¥sqrt{x}}dx=
¥frac{1}{2}¥int_0^3¥frac{1}{¥sqrt{x+1}}dx

〈7-49〉
¥left¥{¥begin{array}{l}
¥displaystyle{
¥int¥frac{x}{¥sqrt{x+1}}dx}=
¥displaystyle{¥frac{2}{3}}(x+1)^{3/2}
-2(x+1)^{1/2}¥equiv J_1(x) ¥¥ ¥¥
¥displaystyle{
¥int¥frac{1}{¥sqrt{x+1}}dx}=
2(x+1)^{1/2}¥equiv J_2(x)
¥end{array}¥right.

〈7-50〉
¥left¥{¥begin{array}{l}
I_1=J_1(3)-J_1(0)
=¥displaystyle{¥frac{8}{3}} ¥¥ ¥¥
I_2=¥displaystyle{
¥frac{1}{2}¥left[J_2(3)-J_2(0)¥right]
=1
¥end{array}¥right.

〈7-51〉
I=I_1+I_2=¥frac{11}{3}

〈7-52〉
I_1=¥int_{C_1}¥vec{A}¥cdot d¥vec{s}

=¥int_0^1xdx+¥int_0^1xdy

〈7-53〉
¥begin{array}{rl}
I_1&=¥displaystyle{
¥int_0^1xdx+¥int_0^1ydy} ¥¥ ¥¥
&=¥displaystyle{
¥left[¥frac{x^2}{2}¥right]_0^1
+¥left[¥frac{y^2}{2}¥right]_0^1=1}
¥end{array}

〈7-54〉
¥begin{array}{rl}
I_2&=¥displaystyle{¥int_{C_2}
¥vec{A}(¥vec{r})¥cdot d¥vec{s}} ¥¥ ¥¥
&=¥displaystyle{¥int_0^1 xdx+¥int_0^1 xdy}
¥end{array}

〈7-55〉
I_2=¥int_0^1 xdx+¥int_0^1 ¥sqrt{y}dy

〈7-56〉
¥begin{array}{rl}
I_2&=¥displaystyle{
¥left[¥frac{x^2}{2}¥right]_0^1+
¥frac{2}{3}¥left[y^{3/2}¥right]_0^1} ¥¥ ¥¥
=¥displaystyle{¥frac{7}{6}}
¥end{array}

〈7-57〉 (7.2.7)
¥begin{array}{rl}
I&=¥displaystyle{
¥int_C¥vec{A}¥cdot d¥vec{s}} ¥¥ ¥¥
&=¥displaystyle{
¥int_C P(x,y)dx+¥int_C Q(x,y)dy}
¥end{array}

〈7-58〉 (7.2.8)
¥frac{¥partial P(x,y)}{¥partial y}=
¥frac{¥partial Q(x,y)}{¥partial x}

〈7-59〉
¥left¥{¥begin{array}{l}
¥displaystyle{¥frac{¥partial P(x,y)}{¥partial y}=2xy
} ¥¥ ¥¥
¥displaystyle{¥frac{¥partial Q(x,y)}{¥partial x}=2xy
}
¥end{array}¥right.

〈7-60〉
¥begin{array}{rl}
I_1&=¥displaystyle{
¥int_{C_1}¥vec{A}¥cdot d¥vec{s}} ¥¥ ¥¥
&=¥displaystyle{
¥int_{C_1} A_xdx+¥int_{C_1} A_ydy} ¥¥ ¥¥
&=¥displaystyle{
¥int_0^1xy^2dx+¥int_0^1x^2ydy}
¥end{array}

〈7-61〉
¥begin{array}{rl}
I_1&=¥displaystyle{
¥int_0^1x^3dx+¥int_0^1y^3dy} ¥¥ ¥¥
&=¥displaystyle{¥left[¥frac{x^4}{4}¥right]_0^1+
¥left[¥frac{y^4}{4}¥right]_0^1
=¥frac{1}{2}}
¥end{array}

〈7-62〉
¥begin{array}{rl}
I_2&=¥displaystyle{
¥int_{C_2}¥vec{A}¥cdot d¥vec{s}} ¥¥ ¥¥
&=¥displaystyle{
¥int_0^1xy^2dx+¥int_0^1x^2ydy}
¥end{array}

〈7-63〉
¥begin{array}{rl}
I_1&=¥displaystyle{
¥int_0^1x^5dx+¥int_0^1y^2dy} ¥¥ ¥¥

&=¥displaystyle{¥left[¥frac{x^6}{6}¥right]_0^1
+¥left[¥frac{y^3}{3}¥right]_0^1=¥frac{1}{2}}
¥end{array}

〈7-64〉　(7.2.9)
¥frac{¥partial P(x,y)}{¥partial y}=
¥frac{¥partial Q(x,y)}{¥partial x}

〈7-65〉　(7.2.10)
df(x,y)=P(x,y)dx+Q(x,y)dy

〈7-66〉　(7.2.11)
¥int_a^b df(x,y)=
¥int_a^b P(x,y)dx+¥int_a^b Q(x,y)dy

〈7-67〉　(6.2.12)
¥begin{array}{l}
f(x_2,y_2)-f(x_1,y_1) ¥¥ ¥¥
¥quad=¥displaystyle{
¥int_a^b P(x.y)dx+¥int_a^b Q(x.y)dy}
¥end{array}

〈7-68〉
¥vec{A}=¥vec{i}x+¥vec{j}x

〈7-69〉
P(x,y)=x

〈7-70〉
Q(x,y)=x

〈7-71〉
¥frac{¥partial P}{¥partial y}=0

〈7-72〉
¥frac{¥partial Q}{¥partial x}=1

〈7-73〉
¥frac{¥partial P}{¥partial y}¥ne
¥frac{¥partial Q}{¥partial x}

〈7-74〉
¥vec{A}=¥vec{i}xy^2+¥vec{j}x^2y

〈7-75〉
P(x,y)=xy^2

〈7-76〉
Q(x,y)=x^2y

〈7-77〉
¥frac{¥partial P}{¥partial y}=2xy

〈7-78〉
¥frac{¥partial Q}{¥partial x}=2xy

〈7-79〉
¥frac{¥partial P}{¥partial y}=
¥frac{¥partial Q}{¥partial x}

〈7-80〉 (7.3.1)
A=¥iint_S¥phi(x,y,z)dS

〈7-81〉
2x+2y+z=2

〈7-82〉
I=¥iint_S(¥vec{r}¥cdot¥vec{n})dS

〈7-83〉
¥vec{r}=¥vec{i}x+¥vec{j}y+¥vec{k}z

〈7-84〉
¥left¥{¥begin{array}{l}
¥vec{a}=-¥vec{i}+¥vec{j} ¥¥ ¥¥
¥vec{b}=-¥vec{j}+2¥vec{k}
¥end{array}¥right.

〈7-85〉
¥begin{array}{rl}

¥vec{a}¥times¥vec{b}
&=(-¥vec{i})¥times(-¥vec{j})+
(-¥vec{i})¥times(2¥vec{k}) ¥¥ ¥¥
&¥quad+(¥vec{j})¥times(-¥vec{j})+
(¥vec{j})¥times(2¥vec{k}) ¥¥ ¥¥
&=¥vec{k}+2¥vec{j}+2¥vec{i}
¥end{array}

<7-86>
¥left|2¥vec{i}+2¥vec{j}+¥vec{k}¥right|=
¥sqrt{2^2+2^2+1^1}=3

<7-87> (7.3.2)
¥vec{n}=¥frac{1}{3}(2¥vec{i}+2¥vec{j}+¥vec{k})

<7-88> (7.3.3)
¥vec{n}(x,y,z)=
¥frac{¥nabla f(x,y,z)}{|¥nabla f(x,y,z)|}

<7-89>
f(x,y,z)=2x+2y+z-2=0

<7-90>
¥begin{array}{l}
¥nabla f(x,y,z) ¥¥ ¥¥
¥quad=¥displaystyle{
¥left(¥vec{i}¥frac{¥partial}{¥partial x}+
¥vec{j}¥frac{¥partial}{¥partial y}+
¥vec{k}¥frac{¥partial}{¥partial z}¥right)}
(2x+2y+z-2) ¥¥ ¥¥
¥quad=2¥vec{i}+2¥vec{j}+¥vec{k}
¥end{array}

<7-91>
¥left|¥nabla f(x,y,z)¥right|=
¥sqrt{2^2+2^2+1^1}=3

<7-92>
¥vec{r}=¥vec{i}x+¥vec{j}y+¥vec{k}z

〈7-93〉
¥begin{array}{rl}
¥vec{r}¥cdot¥vec{n}
&=¥displaystyle{(¥vec{i}x+
¥vec{j}y+
¥vec{k}z)¥cdot¥frac{1}{3}(2¥vec{i}+
2¥vec{j}+¥vec{k})} ¥¥ ¥¥
&=¥displaystyle{¥frac{2x+2y+z}{3}}
¥end{array}

〈7-94〉
2x+2y+z=2

〈7-95〉
Z=2-2x-2y

〈7-96〉
¥vec{r}¥cdot¥vec{n}=¥frac{2}{3}

〈7-97〉
¥sqrt{(x_1-x_2)^2+(y_1-y_2)^2+
(z_1-z_2)^2}

〈7-98〉
I=¥frac{2}{3}¥times¥frac{3}{2}=1

〈7-99〉 (7.3.4)
¥int_C¥vec{A}(¥vec{r})¥cdot d¥vec{s}=
¥iint_S
¥left[¥mbox{rot}¥vec{A}(¥vec{r})¥right]_ndS

〈7-100〉
¥displaystyle{
¥int_A^B¥vec{A}(¥vec{r})¥cdot d¥vec{s}=
I_C(A¥rightarrow B)}

〈7-101〉
I_{C_1}(A¥rightarrow B)=I_{C_2}(A¥rightarrow B)

〈7-102〉

I_{C_1}(A¥rightarrow B)=-I_{C_2}(B¥rightarrow A)

〈7-103〉
I_{C_1}(A¥rightarrow B)+I_{C_2}(B¥rightarrow A)=0

〈7-104〉
¥displaystyle{¥oint_C¥vec{A}(¥vec{r})¥cdot d¥vec{s}=
0}

〈7-105〉
¥displaystyle{
¥iint_S¥left[¥vec{A}(¥vec{r})¥right]_ndS=0}

〈7-106〉
¥mbox{rot}¥vec{A}(¥vec{r})=0

〈7-107〉
¥mbox{rot}¥cdot¥mbox{grad}f(¥vec{r})=0

〈7-108〉
¥vec{A}(¥vec{r})=¥mbox{grad}f(¥vec{r})

〈7-109〉 (7.3.5)
I=¥iint_{S}f(x,y)dxdy

〈7-110〉
x_1¥le x¥le x_2

〈7-111〉
y_1¥le y¥le y_2

〈7-112〉 (7.3.6)
I=¥int_{x_1}^{x_2}dx¥int_{y_1}^{y_2}dyf(x,y)

〈7-113〉
¥frac{d}{dx}f(x)

〈7-114〉
¥frac{df(x)}{dx}

```
<7-115>
¥int_{x_1}^{x_2}dxf(x)

<7-116>
¥int_{x_1}^{x_2}f(x)dx

<7-117>  (7.3.7)
¥left¥{¥begin{array}{l}
x=r¥cos¥theta ¥¥ ¥¥
y=r¥sin¥theta
¥end{array}¥right.

<7-118>  (7.3.8)
I=¥int_{-¥infty}^{+¥infty}dx
¥int_{-¥infty}^{+¥infty}dyf(x,y)

<7-119>  (7.3.9)
I=¥int_{r_1}^{r_2}dr
¥int_{¥theta_1}^{¥theta_2}d¥theta
|J|f(r¥cos¥theta,r¥sin¥theta )

<7-120>  (7.3.10)
¥left(¥begin{array}{cc}
¥displaystyle{
¥frac{¥partial x}{¥partial r}} &
¥displaystyle{
¥frac{¥partial x}{¥partial ¥theta}} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial y}{¥partial r}} &
¥displaystyle{¥frac{
¥partial y}{¥partial ¥theta}}
¥end{array}¥right)

<7-121>
¥left(¥begin{array}{cc}
¥displaystyle{
¥frac{¥partial x}{¥partial r}} &
¥displaystyle{
¥frac{¥partial x}{¥partial ¥theta}} ¥¥ ¥¥
¥displaystyle{
```

```
¥frac{¥partial y}{¥partial r}} &
¥displaystyle{
¥frac{¥partial y}{¥partial ¥theta}}
¥end{array}¥right)=
¥left(¥begin{array}{cc}
¥cos¥theta & -r¥sin¥theta ¥¥ ¥¥
¥sin¥theta &
r¥cos¥theta
¥end{array}¥right)
```

〈7-122〉

```
¥begin{array}{rl}
J&=¥left|¥begin{array}{cc}
¥cos¥theta & -r¥sin¥theta ¥¥ ¥¥
¥sin¥theta &
r¥cos¥theta
¥end{array}¥right| ¥¥ ¥¥
&=r¥cos^2¥theta+r¥sin^2¥theta ¥¥ ¥¥
&=r
¥end{array}
```

〈7-123〉

```
¥left(¥begin{array}{l}
x=-¥infty¥sim+¥infty ¥¥ ¥¥
y=-¥infty¥sim+¥infty
¥end{array}¥right)¥quad¥rightarrow¥quad
¥left(¥begin{array}{l}
r=0¥sim+¥infty ¥¥ ¥¥
¥theta=0¥sim2¥pi
¥end{array}¥right)
```

〈7-124〉　(7.3.11)

```
I=¥int_{0}^{¥infty}dr¥int_{0}^{2¥pi}d¥theta
rf(r¥cos¥theta,r¥sin¥theta)
```

〈7-125〉　(7.3.12)

```
¥int_{-¥infty}^{+¥infty}e^{-x^2}dx
=¥sqrt{¥pi}
```

〈7-126〉

```
¥begin{array}{rl}
I^2&=¥left(¥int_{-¥infty}^{+¥infty}
e^{-x^2}dx¥right)¥times
¥left(¥int_{-¥infty}^{+¥infty}
e^{-y^2}dy¥right) ¥¥ ¥¥
&=¥int_{-¥infty}^{+¥infty}dx
¥int_{-¥infty}^{+¥infty}dy
e^{-(x^2+y^2)}
¥end{array}

<7-127>
x^2+y^2=r^2

<7-128>
¥begin{array}{rl}
I^2&=¥displaystyle{
¥left(¥int_{0}^{¥infty}e^{-r^2}rdr¥right)
¥left(¥int_{0}^{2¥pi}d¥theta¥right)} ¥¥ ¥¥
&=¥displaystyle{
2¥pi¥int_{0}^{¥infty}e^{-r^2}rdr}
¥end{array}

<7-129>
J=¥frac{dr}{ds}=¥frac{1}{2¥sqrt{s}}

<7-130>
¥begin{array}{rl}
I^2&=¥displaystyle{
2¥pi¥int_{0}^{¥infty}
e^{-s}|J|¥sqrt{s}ds} ¥¥ ¥¥
&=¥displaystyle{
¥pi¥int_{0}^{¥infty}e^{-s}ds}
¥end{array}

<7-131>
¥int e^{-s}ds=-e^{-s}

<7-132>
¥begin{array}{rl}
I^2&=¥displaystyle{
```

-¥pi¥left[e^{-s}¥right]_0^¥infty} ¥¥ ¥¥
&=¥pi
¥end{array}

〈7-133〉
I=¥sqrt{¥pi}

〈7-134〉
¥lim_{R¥rightarrow¥infty}¥int_{0}^{R}rdr
¥int_{0}^{2¥pi}d¥theta f¥equiv
¥lim_{R¥rightarrow¥infty}I(R)

〈7-135〉
¥int_{0}^{R}rdr=¥frac{R^2}{2}

〈7-136〉
¥int_{0}^{2¥pi}d¥theta=2¥pi

〈7-137〉
I(R)=¥pi R^2

〈7-138〉 (7.4.1)
I=¥iiint_V¥phi(x,y,z)dV

〈7-139〉 (7.4.2)
¥iint_S¥vec{E}(¥vec{r})¥cdot¥vec{n}(¥vec{r})dS=
¥iiint_V¥mbox{div}¥vec{E}(¥vec{r})dV

〈7-140〉
¥left¥{¥begin{array}{l}
x=r¥sin¥theta¥cos¥phi ¥¥ ¥¥
y=r¥sin¥theta¥sin¥phi ¥¥ ¥¥
z=r¥cos¥theta
¥end{array}¥right.

〈7-141〉 (7.4.4)
¥begin{array}{rl}
I&=¥displaystyle{¥int_{-¥infty}^{+¥infty}dx
¥int_{-¥infty}^{+¥infty}dy
¥int_{-¥infty}^{+¥infty}dzf(x,y,z)} ¥¥ ¥¥

&=¥displaystyle{¥int_{r_1}^{r_2}dr
¥int_{¥theta_1}^{¥theta_2}d¥theta
¥int_{¥phi_1}^{¥phi_2}d¥phi
|J|¥tilde{f}(r,¥theta,¥phi)}
¥end{array}

〈7-142〉 (7.4.5)
J=¥left|
¥begin{array}{ccc}
¥displaystyle{
¥frac{¥partial x}{¥partial r}} &
¥displaystyle{
¥frac{¥partial x}{¥partial¥theta}} &
¥displaystyle{
¥frac{¥partial x}{¥partial¥phi}} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial y}{¥partial r} &
¥displaystyle{
¥frac{¥partial y}{¥partial¥theta}} &
¥displaystyle{
¥frac{¥partial y}{¥partial¥phi}} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial z}{¥partial r}} &
¥displaystyle{
¥frac{¥partial z}{¥partial¥theta}} &
¥displaystyle{
¥frac{¥partial z}{¥partial¥phi}}
¥end{array}¥right|

〈7-143〉
¥begin{array}{l}
¥left¥{¥begin{array}{l}
¥displaystyle{
¥frac{¥partial x}{¥partial r}=
¥sin¥theta¥cos¥phi} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial x}{¥partial¥theta}=
r¥cos¥theta¥cos¥phi} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial x}{¥partial¥theta}=

```
-r¥sin¥theta¥sin¥phi}
¥end{array}¥right. ¥¥ ¥¥
¥left¥{¥begin{array}{l}
¥displaystyle{
¥frac{¥partial y}{¥partial r}=
¥sin¥theta¥sin¥phi} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial x}{¥partial¥theta}=
r¥cos¥theta¥sin¥phi} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial y}{¥partial¥theta}=
r¥sin¥theta¥cos¥phi}
¥end{array}¥right. ¥¥ ¥¥
¥left¥{¥begin{array}{l}
¥displaystyle{
¥frac{¥partial z}{¥partial r}=
¥cos¥theta} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial z}{¥partial¥theta}=
-r¥sin¥theta} ¥¥ ¥¥
¥displaystyle{
¥frac{¥partial z}{¥partial¥theta}=0}
¥end{array}¥right.
¥end{array}
```

<7-144>

```
¥begin{array}{rl}
J&=¥left|¥begin{array}{ccc}
¥sin¥theta¥cos¥phi
& r¥cos¥theta¥cos¥phi
& -r¥sin¥theta¥sin¥phi ¥¥ ¥¥
¥sin¥theta¥sin¥phi
& r¥cos¥theta¥sin¥phi
& r¥sin¥theta¥cos¥phi ¥¥ ¥¥
¥cos¥theta
& -r¥sin¥theta & 0
¥end{array}¥right| ¥¥ ¥¥
&=r^2¥sin¥theta
¥end{array}
```

〈7-145〉
¥left¥{¥begin{array}{l}
x=-¥infty¥sim+¥infty ¥¥ ¥¥
y=-¥infty¥sim+¥infty ¥¥ ¥¥
z=-¥infty¥sim+¥infty
¥end{array}¥right¥}¥Rightarrow
¥left¥{¥begin{array}{l}
r=0¥sim+¥infty ¥¥ ¥¥
¥theta=-¥pi¥sim¥pi ¥¥ ¥¥
¥phi=0¥sim2¥pi
¥end{array}¥right¥}

〈7-146〉 (7.4.6)
I=¥int_{0}^{¥infty}r^2dr
¥int_{-¥pi}^{¥pi}¥sin¥theta d¥theta
¥int_{0}^{2¥pi}d¥phi¥tilde{f}(r,¥theta,¥phi)

〈7-147〉
¥int_{0}^{R}r^2dr
¥int_{-¥pi}^{¥pi}¥sin¥theta d¥theta
¥int_{0}^{2¥pi}d¥phi
=¥frac{4¥pi}{3}R^3

〈8-1〉 (8.1.1)
¥left¥{¥begin{array}{l}
x=r¥cos¥theta ¥¥ ¥¥ y=r¥sin¥theta
¥end{array}¥right.

〈8-2〉 (8.1.2)
z=re^{i¥theta}

〈8-3〉
¥frac{f(z)}{z-z_0}

〈8-4〉
I=¥oint_C¥frac{f(z)}{z-z_0}dz

〈8-5〉 (8.1.3)
¥frac{1}{2¥pi i}¥oint_C¥frac{f(z)}{z-z_0}dz
=f(z_0)

〈8-6〉　(8.1.4)
¥int_{-¥infty}^{¥infty}¥frac{1}{x^2+1}dx
=¥pi

〈8-7〉　(8.1.5)
¥begin{array}{rl}
I(X)&=¥displaystyle{
¥int_{-X}^{+X}¥frac{1}{x^2+1}dx}+
¥displaystyle{
¥int_{C'}¥frac{1}{z^2+1}dz} ¥¥ ¥¥
&=¥displaystyle{
¥int_{C(X)}¥frac{1}{z^2+1}dz}
¥end{array}

〈8-8〉
x^2+a^2=(x+ia)(x-ia)

〈8-9〉
z^2+1=(z+i)(z-i)

〈8-10〉　(8.1.6)
¥begin{array}{rl}
I(X)&=¥displaystyle{
¥int_{C(X)}¥frac{1}{(z+i)(z-i)}dz} ¥¥ ¥¥
&=¥displaystyle{
¥int_{C(X)}¥frac{f(z)}{z-i}dz}
¥end{array}

〈8-11〉
f(z)=¥frac{1}{z+i}

〈8-12〉
¥sqrt{(-i)¥times(-i)^*}=¥sqrt{-i¥times i}
=1

〈8-13〉　(8.1.7)
¥begin{array}{rl}
I(X)&=2¥pi i¥times f(z=i) ¥¥ ¥¥
&=2¥pi i¥times¥displaystyle{

```
¥frac{1}{2i}}=¥pi
¥end{array}
```

〈8-14〉 (8.1.8)

```
¥begin{array}{rl}
I(X)&=¥displaystyle{
¥int_{-X}^{X}¥frac{1}{x^2+1}dx+
¥int_{C'}¥frac{1}{z^2+1}dz} ¥¥ ¥¥
&¥rightarrow¥displaystyle{
¥int_{-¥infty}^{¥infty}¥frac{1}{x^2+1}dx}
¥end{array}
```

〈8-15〉

```
¥int_{-¥infty}^{¥infty}¥frac{1}{x^2+1}dx
=¥pi
```

〈8-16〉

```
¥int_{-¥infty}^{¥infty}¥frac{1}{x^2+2}dx
=¥frac{¥pi}{¥sqrt{2}}
```

〈8-17〉

```
¥int_{-¥infty}^{¥infty}¥frac{1}{x^2+2x+3}dx
=¥frac{¥pi}{¥sqrt{2}}
```

〈8-18〉

```
¥int_{0}^{¥infty}¥frac{¥cos x}{x^2+1}dx
=¥frac{¥pi}{¥sqrt{2e}}
```

〈8-19〉

```
¥int_{0}^{¥infty}¥frac{1}{x^3+1}dx
=¥frac{2¥pi}{3¥sqrt{3}}
```

〈8-20〉

```
y=A_1¥sin¥left(¥frac{¥pi}{¥ell}x¥right)
¥equiv y_1
```

〈8-21〉

```
y=A_2¥sin¥left(¥frac{2¥pi}{¥ell}x¥right)
¥equiv y_2
```

〈8-22〉
y=A_3¥sin¥left(¥frac{3¥pi}{¥ell}x¥right)
¥equiv y_3

〈8-23〉　(8.2.1)
y_n(x)=A_n¥sin¥left(¥frac{n¥pi}{¥ell}x¥right),
¥quad(n=1,2,3,¥cdots)

〈8-24〉
¥lambda_n=¥frac{2¥ell}{n},
¥quad(n=1,2,3,¥cdots)

〈8-25〉　(8.2.2)
y_n(x)=
A_n¥sin¥left(¥frac{2¥pi x}{¥lambda_n}¥right),
¥quad(n=1,2,3,¥cdots)

〈8-26〉　(8.2.3)
¥begin{array}{rl}
f(x)&=¥displaystyle{
¥sum_{n=1}^{¥infty}F_ny_n(x)} ¥¥ ¥¥
&=¥displaystyle{
¥sum_{n=1}^{¥infty}F_n
¥sin¥left(¥frac{2¥pi x}{¥lambda_n}¥right)}
¥end{array}

〈8-27〉　(8.2.4)
¥sum_{n=1}^{¥infty}S_n=S_1+S_2+¥cdots

〈8-28〉
¥begin{array}{ll}
¥mbox{(*)}&¥quad¥displaystyle{¥int_{0}^{¥ell}
¥sin¥left(¥frac{m¥pi}{¥ell}x¥right)f(x)dx}
¥¥ ¥¥
&¥quad=¥displaystyle{¥sum_{n=1}^{¥infty}F_n
¥int_{0}^{¥ell}¥sin¥left(¥frac{m¥pi}{¥ell}x¥right)
¥sin¥left(¥frac{n¥pi}{¥ell}x¥right)dx}
¥end{array}

〈8-29〉　(8.2.5)

```
¥left¥{¥begin{array}{l}
¥sin(A¥pm B)=¥sin A¥cos B¥pm¥cos A¥sin B ¥¥ ¥¥
¥cos(A¥pm B)=¥cos A¥cos B¥mp¥sin A¥sin B
¥end{array}¥right.
```

<8-30>

```
¥sin A¥sin B=¥frac{1}{2}[¥cos(A-B)-¥cos(A+B)]
```

<8-31>

```
¥begin{array}{l}
¥displaystyle{¥int_{0}^{¥ell}
¥sin¥left(¥frac{m¥pi}{¥ell}x¥right)
¥sin¥left(¥frac{n¥pi}{¥ell}x¥right)dx} ¥¥ ¥¥
&¥quad=¥displaystyle{¥frac{1}{2}¥int_{0}^{¥ell}
¥left[¥cos¥left(¥frac{(m-n)¥pi}{¥ell}x¥right)
-¥cos¥left(¥frac{(m+n)¥pi}{¥ell}x¥right)¥right]dx}
¥end{array}
```

<8-32>

```
¥begin{array}{rl}
¥mbox{(**)}&¥quad¥displaystyle{¥int_{0}^{¥ell}
¥sin¥left(¥frac{m¥pi}{¥ell}x¥right)f(x)dx} ¥¥ ¥¥
&¥quad=¥displaystyle{
¥frac{1}{2}¥sum_{n=1}^{¥infty}F_n
¥left[¥int_{0}^{¥ell}
¥cos¥left(¥frac{(m-n)¥pi}{¥ell}x¥right)dx
¥right.}¥¥ ¥¥
&¥quad¥quad¥left.-¥displaystyle{
¥int_{0}^{¥ell}
¥cos¥left(¥frac{(m+n)¥pi}{¥ell}x¥right)dx
¥right]} ¥¥ ¥¥
&¥quad=¥displaystyle{
¥frac{1}{2}¥sum_{n=1}^{¥infty}F_n
¥left[¥frac{¥ell}{(m-n)¥pi}
¥sin¥left(¥frac{(m-n)¥pi}{¥ell}x¥right)
¥right.}¥¥ ¥¥
&¥quad¥quad¥left.-¥displaystyle{
¥frac{¥ell}{(m+n)¥pi}
¥sin¥left(¥frac{(m+n)¥pi}{¥ell}x¥right)
¥right]_{x=0}^{x=¥ell}}}
```

¥end{array}

〈8-33〉
¥int_{0}^{¥ell}dx=¥ell

〈8-34〉
¥lim_{n¥rightarrow m}¥left[¥frac{¥ell}{(m-n)¥pi}
¥sin¥left(¥frac{(m-n)¥pi}{¥ell}¥right)¥right]

¥begin{array}{l}
¥displaystyle{
¥int_{0}^{¥ell}¥sin¥left(¥frac{m¥pi}{¥ell}x¥right)
¥sin¥left(¥frac{n¥pi}{¥ell}x¥right)dx} ¥¥ ¥¥
¥quad=¥left¥{¥begin{array}{ll}
¥displaystyle{
¥frac{¥ell}{2}}, & ¥mbox{(if $n=m$).} ¥¥ ¥¥
0, & ¥mbox{(if $n¥ne m$).}
¥end{array}¥right.
¥end{array}

〈8-36〉 (8.2.7)
¥begin{array}{l}
¥displaystyle{F_m=¥frac{2}{¥ell}¥int_{0}^{¥ell}
¥sin¥left(¥frac{m¥pi}{¥ell}x¥right)f(x)dx}, ¥¥ ¥¥
¥quad (¥mbox{where m is a natural number.})
¥end{array}

〈8-37〉 (8.2.8)
f(x)=¥sum_{n=1}^{¥infty}
F_n¥sin¥left(¥frac{n¥pi}{¥ell}x¥right)

〈8-38〉 (8.2.9)
F_n=¥frac{2}{¥ell}¥int_{0}^{¥ell}
¥sin¥left(¥frac{n¥pi}{¥ell}x¥right)f(x)dx

〈8-39〉 (8.2.10)
u_n(x)=¥sqrt{¥frac{2}{¥ell}}
¥sin¥left(¥frac{n¥pi}{¥ell}x¥right),
¥quad(n=1,2,¥cdots)

<8-40> (8.2.11)
f(x)=¥sqrt{¥frac{¥ell}{2}}¥sum_{n=1}^{¥infty}
F_nu_n(x)

<8-41>
F_n=¥sqrt{¥frac{2}{¥ell}}¥int_{0}^{¥ell}
u_n(x)f(x)dx

<8-42> (8.2.12)
¥int_{0}^{¥ell}u_n(x)u_m(x)dx=¥left¥{
¥begin{array}{ll}
1 & ¥mbox{when $n=m$.} ¥¥ ¥¥
0 & ¥mbox{when $n¥ne m$.}
¥end{array}¥right.

<8-43> (8.2.13)
f(x)=F_0+¥sum_{n=1}^{¥infty}
¥left[F_n¥sin¥left(¥frac{n¥pi}{¥ell}x¥right)+
G_n¥cos¥left(¥frac{n¥pi}{¥ell}x¥right)¥right]

<8-44> (8.2.14)
¥left¥{¥begin{array}{l}
¥displaystyle{
F_0=¥frac{1}{¥ell}¥int_{0}^{¥ell}f(x)dx} ¥¥ ¥¥
¥displaystyle{
F_n=¥frac{2}{¥ell}¥int_{0}^{¥ell}
¥sin¥left(n¥pi}{¥ell}x¥right)f(x)dx} ¥¥ ¥¥
¥displaystyle{
G_n=¥frac{2}{¥ell}¥int_{0}^{¥ell}
¥cos¥left(n¥pi}{¥ell}x¥right)f(x)dx}
¥end{array}¥right.

<8-45> (8.2.15)
¥left¥{¥begin{array}{l}
¥displaystyle{
F_0=¥frac{1}{L}¥int_{-L/2}^{L/2}f(x)dx} ¥¥ ¥¥
¥displaystyle{
F_n=¥frac{2}{L}¥int_{-L/2}^{L/2}
¥sin¥left(¥frac{n¥pi}{L}x¥right)f(x)dx} ¥¥ ¥¥

¥displaystyle{
G_n=¥frac{2}{L}¥int_{-L/2}^{L/2}
¥cos¥left(¥frac{n¥pi}{L}x¥right)f(x)dx}
¥end{array}¥right.

<8-46>
f(-x)=+f(x)

<8-47>
f(-x)=-f(x)

<8-48>
¥frac{dx}{dy}=-1

<8-49>
¥int_{b}^{a}f(x)dx=-¥int_{a}^{b}f(x)

<8-50>
¥sin(-¥theta)=-¥sin¥theta

<8-51>
¥left¥{¥begin{array}{rl}
F_n&=¥displaystyle{
¥frac{2}{L}¥int_{L/2}^{-L/2}
¥sin¥left(-¥frac{n¥pi}{L}y¥right)
f(-y)(-1)dy} ¥¥ ¥¥
&=¥displaystyle{
-¥frac{2}{L}¥int_{-L/2}^{L/2}
¥sin¥left(¥frac{n¥pi}{L}y¥right)
f(y)dy} ¥¥ ¥¥
&=-F_n
¥end{array}¥right.

<8-52> (8.2.16)
f(x)=¥frac{1}{2¥pi}¥int_{-¥infty}^{¥infty}
g(k)e^{ikx}dk

<8-53> (8.2.17)
g(k)=¥int_{-¥infty}^{¥infty}e^{-ikx}f(x)dx

〈8-54〉　(8.2.18)
```
e^{¥pm ikx}=¥cos(kx)¥pm i¥sin(kx)
```

〈8-55〉
```
|x|=¥left¥{¥begin{array}{ll}
+x, & ¥mbox{when $x¥ge0$.} ¥¥ ¥¥
-x, & ¥mbox{when $x¥le0$.}
¥end{array}¥right.
```

〈8-56〉
```
¥begin{array}{rl}
g(k)&=¥displaystyle{
¥int_{-¥infty}^{¥infty}
e^{-ikx}e^{-a|x|}dx}} ¥¥ ¥¥
&=¥displaystyle{
¥int_{-¥infty}^{0}e^{-ikx}e^{-a|x|}dx}}
+¥displaystyle{
¥int_{0}^{¥infty}e^{-ikx}e^{-a|x|}dx}}
¥end{array}¥right.
```

〈8-57〉
```
¥begin{array}{rl}
g(k)&=¥displaystyle{¥int_{-¥infty}^{¥infty}
e^{-ikx}e^{-a|x|}dx}} ¥¥ ¥¥
&=¥displaystyle{
¥int_{-¥infty}^{0}e^{-ikx}e^{ax}dx}}
+¥displaystyle{
¥int_{0}^{¥infty}e^{-ikx}e^{-ax}dx}} ¥¥ ¥¥
&=¥displaystyle{
¥int_{-¥infty}^{0}e^{(a-ik)x}dx}}
+¥displaystyle{
¥int_{0}^{¥infty}e^{(-a-ik)x}dx}}
¥end{array}¥right.
```

〈8-58〉
```
¥int e^{px}dx=¥frac{e^{px}}{p}
```

〈8-59〉
```
g(k)=¥left[
¥frac{e^{(a-ik)x}}{a-ik}
```

¥right]_{-¥infty}^{0}
+¥left[
¥frac{e^{(-a-ik)x}}{-a-ik}
¥right]_{0}^{¥infty}

＜8-60＞
g(k)=¥frac{1}{a-ik}-¥frac{1}{-a-ik}
=¥frac{2a}{a^2+k^2}

＜8-61＞
g(k)=¥frac{¥sqrt{¥pi}}{a}e^{-k^2/(4a^2)}

＜8-62＞
F(s)=¥int_{0}^{¥infty}f(x)e^{-sx}dx

＜8-63＞　(8.3.2)
Z(¥beta)=¥int_{0}^{¥infty}
g(E)e^{-¥beta E}dE

＜8-64＞
¥mbox{any constant a}

＜8-65＞
¥frac{a}{s}

＜8-66＞
x^n¥quad(n>0)

＜8-67＞
¥frac{n!}{s^{n+1}}

＜8-68＞
e^{-¥lambda x}¥quad
¥mbox{with a constant $¥lambda$.}

＜8-69＞
¥frac{1}{s+¥lambda}

＜8-70＞
¥sin(¥lambda x)¥quad

¥mbox{with a constant $¥lambda$.}

〈8-71〉
¥frac{¥lambda}{s^2+¥lambda^2}

〈8-72〉
¥cos(¥lambda x)¥quad
¥mbox{with a constant $¥lambda$.}

〈8-73〉
¥frac{s}{s^2+¥lambda^2}

〈8-74〉 (8.3.3)
L¥left¥{af(x)+bg(x)¥right¥}=
aL¥left¥{f(x)¥right¥}
+bL¥left¥{g(x)¥right¥}

〈9-1〉
J[f]=¥int_{0}^{1}f(x)dx

〈9-2〉
J=¥int_{0}^{1}xdx
=¥left[¥frac{1}{2}x^2¥right]_{0}^{1}
=¥frac{1}{2}

〈9-3〉
J=¥int_{0}^{1}x^2dx
=¥left[¥frac{1}{3}x^3¥right]_{0}^{1}
=¥frac{1}{3}

〈9-4〉
J=¥int_{0}^{1}x^3dx
=¥left[¥frac{1}{4}x^4¥right]_{0}^{1}
=¥frac{1}{4}

〈9-5〉
J[f]=¥int_{0}^{1}F[f(x)]dx

〈9-6〉
F[f(x)]=f(x)+1

〈9-7〉
J=¥int_{0}^{1}(x+1)dx
=¥left[¥frac{1}{2}x^2+x¥right]_{0}^{1}
=¥frac{3}{2}

〈9-8〉
J=¥int_{0}^{1}(x^2+1)dx
=¥left[¥frac{1}{3}x^3+x¥right]_{0}^{1}
=¥frac{4}{3}

〈9-9〉
J=¥int_{0}^{1}(x^3+1)dx
=¥left[¥frac{1}{4}x^4+x¥right]_{0}^{1}
=¥frac{5}{4}

〈9-10〉 (9.1.1)
J[f]=¥int_{A}^{B}F[f(x)]dx

〈9-11〉 (9.1.2)
J[f+¥Delta]-J[f]
=¥int_{A}^{B}F[f(x)+¥Delta(x)]dx
-¥int_{A}^{B}F[f(x)]dx

〈9-12〉 (9.1.3)
¥delta J[f]=
¥int_{A}^{B}¥left(
¥frac{¥partial F}{¥partial f}¥delta f
¥right)dx

〈9-13〉 (9.1.4)
¥frac{d}{dx}¥frac{¥partial F}{¥partial f'}
-¥frac{¥partial F}{¥partial f}=0

〈9-14〉
s=¥sqrt{(x_2-x_1)^2+(y_2-y_1)^2}

〈9-15〉
¥Delta s=¥sqrt{(¥Delta x)^2+(¥Delta y)^2}
=¥sqrt{1+¥left(¥frac{¥Delta y}{¥Delta x}¥right)^2}

¥Delta x

〈9-16〉
L=¥int_{0}^{a}¥sqrt{1+y'^2}dx

〈9-17〉　(9.1.6)
¥frac{dF[y'(x)]}{dy'}
=¥frac{d¥sqrt{1+{y'}^2}}{d{y'}}
=¥frac{y'}{¥sqrt{1+{y'}^2}}

〈9-18〉　(9.1.7)
¥frac{d}{dx}¥frac{y'}{¥sqrt{1+{y'}^2}}=0

〈9-19〉　(9.1.8)
y'=A

〈9-20〉　(9.1.9)
y=Ax+B

〈9-21〉　(9.1.10)
¥left¥{¥begin{array}{l}
0=B ¥¥ ¥¥ b=Aa
¥end{array}¥right.

〈9-22〉　(9.1.11)
B=0 ¥¥ ¥¥ ¥displaystyle{A=¥frac{b}{a}}
¥end{array}¥right.

〈9-23〉　(9.1.12)
y=¥frac{b}{a}x

〈10-1〉　(10.1.1)
¥int_{-¥infty}^{+¥infty}¥delta(x)f(x)dx=f(0)

〈10-2〉　(10.1.2)
¥int_{-¥infty}^{+¥infty}¥delta(x-a)f(x)dx=f(a)

〈10-3〉　(10.1.3)
¥delta(x)
=¥lim_{h¥rightarrow0}¥frac{1}{¥sqrt{2¥pi}h}

e^{-x^2/(2h^2)}

＜10-4＞　(10.1.4)
¥delta(x)=¥lim_{h¥rightarrow0}¥frac{1}{¥pi}
¥frac{h}{x^2+h^2}

＜10-5＞　(10.1.5)
¥delta(x)=¥lim_{n¥rightarrow¥infty}
¥frac{¥sin(nx)}{¥pi x}

＜10-6＞　(10.1.6)
¥int_{-¥infty}^{+¥infty}¥delta(x)dx=1