

<1-1> (1.1.1)

$$\frac{dx(t)}{dt} \equiv v(t)$$

<1-2> (1.1.2)

$$s(t) = |v(t)|$$

<1-3> (1.1.3)

$$x(t) = \int v(t) dt + C_1$$

<1-4> (1.1.4)

$\begin{equation*}$

$$\frac{dv(t)}{dt} \equiv a(t)$$

$\end{equation*}$

<1-5> (1.1.5)

$$v(t) = \int a(t) dt + C_2$$

<2-1> (2.1.1)

$$\vec{a} = \frac{\vec{F}}{m}$$

<2-2> (2.1.2)

$$\vec{F}_{21} = -\vec{F}_{12} \quad \text{or} \quad \vec{F}_{21} + \vec{F}_{12} = 0$$

<2-3> (2.1.3)

$$m \frac{d^2 x(t)}{dt^2} = F$$

<2-4> (2.2.1)

$$m \frac{d^2 x(t)}{dt^2} = 0$$

<2-5> (2.2.2)

$$m \frac{dv(t)}{dt} = 0$$

<2-6>

$$v(t) = C_2$$

<2-7> (2.2.3)

$$v(t) = v_0$$

<2-8>

$$x(t) = \int v_0 dt + C_1$$

<2-9> (2.2.4)

$$x(t) = v_0 t + x_0$$

<2-10> (2.2.5)

$$m \frac{d^2 x(t)}{dt^2} = -mg$$

<2-11> (2.2.6)

$$v(t) = v_0 - \int g dt = v_0 - gt$$

<2-12> (2.2.7)

$\begin{array}{l} \end{array}$

$$x(t) = \int v(t) dt = x_0 + \int v(t) dt$$

$$= \int (v_0 - gt) dt = x_0 + v_0 t - \frac{1}{2} g t^2$$

$$= x_0 + v_0 t - \frac{1}{2} g t^2$$

\end{array}

<2-13> (2.2.8)

$\left[\begin{array}{l} \end{array} \right]$

$$v(t) = v_0 - gt$$

$$x(t) = x_0 + \int v(t) dt = x_0 + v_0 t - \frac{1}{2} g t^2$$

$\end{array} \right]$

<2-14>

$$m_1 \frac{d^2 x_1(t)}{dt^2} = F_{12}$$

<2-15>

$$m_2 \frac{d^2 x_2(t)}{dt^2} = F_{21}$$

<2-16> (2.3.1)

$$F_{12}(x) = -F_{21}(x)$$

<2-17> (2.4.1)

$$m_1 \frac{d^2 x_1(t)}{dt^2} + m_2 \frac{d^2 x_2(t)}{dt^2}$$

$$= \frac{d^2 [m_1 x_1(t) + m_2 x_2(t)]}{dt^2} = 0$$

<2-18> (2.4.2)

$$X(t) = \frac{m_1 x_1(t) + m_2 x_2(t)}{m_1 + m_2}$$

<2-19> (2.4.3)

$$Q = \frac{w_1 Q_1 + w_2 Q_2}{w_1 + w_2}$$

<2-20> (2.4.4)

$$M \frac{d^2 X(t)}{dt^2} = 0$$

<2-21> (2.4.5)

$$m \frac{dv(t)}{dt} = F$$

<2-22> (2.4.6)

$$\frac{d[mv(t)]}{dt} = F$$

<2-23> (2.4.7)

$$mv(t) \equiv p(t)$$

<2-24> (2.4.8)

$$\frac{dp(t)}{dt} = F$$

<2-25> (2.4.9)

$$\begin{array}{l} \frac{dp_1(t)}{dt} = F_{11} \\ \frac{dp_2(t)}{dt} = F_{12} \end{array}$$

<2-26> (2.4.10)

$$V(t) = \frac{dX(t)}{dt}$$

<2-27> (2.4.11)

$$M \frac{dV(t)}{dt} = 0$$

<2-28> (2.4.12)

$$\frac{dP(t)}{dt} = 0$$

<2-29>

$$\begin{array}{l} V(t) = \frac{d}{dt} \left(\frac{m_1 x_1(t) + m_2 x_2(t)}{m_1 + m_2} \right) \\ = \frac{1}{M} \frac{d}{dt} (m_1 \dot{x}_1 + m_2 \dot{x}_2) \\ = \frac{p_1}{M} + \frac{p_2}{M} \end{array}$$

<2-30> (2.4.13)

$$P = p_1 + p_2$$

<2-31> (2.5.1)

$$m \frac{d^2 x}{dt^2} = m \frac{dv}{dt} = F(x)$$

<2-32> (2.5.2)

$$\frac{df(y(x))}{dx} = \frac{df(y)}{dy} \frac{dy(x)}{dx}$$

<2-33> (2.5.3)

$$T(v) = \frac{1}{2} m v^2$$

<2-34>

$$\begin{array}{l} \frac{dT}{dt} = \frac{dT}{dv} \frac{dv}{dt} \\ = \frac{1}{2} m \frac{dv^2}{dv} \frac{dv}{dt} \end{array}$$

$$\frac{d}{dt} \left(m \frac{dv}{dt} \right) = \frac{d}{dt} (mv)$$

<2-35>

$$\frac{dT}{dt} = F(x) \frac{dx}{dt}$$

<2-36>

$$\int_{t_1}^{t_2} \frac{dT}{dt} dt = \int_{t_1}^{t_2} F(x) \frac{dx}{dt} dt$$

<2-37>

$$\int_{T_1}^{T_2} dT = \int_{x_1}^{x_2} F(x) dx$$

$$T_2 - T_1 = \int_{x_1}^{x_2} F(x) dx$$

<2-38>

$$\int_{t_1}^{t_2} F(x) \frac{dx}{dt} dt = \int_{x_1}^{x_2} F(x) dx$$

<2-39> (2.5.4)

$$T_2 - T_1 = \int_{x_1}^{x_2} F(x) dx$$

<2-40> (2.5.5)

$$\int_{x_1}^{x_2} F(x) dx = - \int_{x_2}^{x_1} F(x) dx$$

<2-41> (2.5.6)

$$F(x) = - \frac{dV(x)}{dx}$$

<2-42> (2.5.7)

$$\int_{x_1}^{x_2} F(x) dx = - \int_{x_1}^{x_2} \frac{dV(x)}{dx} dx = V(x_1) - V(x_2)$$

<2-43> (2.5.8)

$$T_1 + V(x_1) = T_2 + V(x_2)$$

<2-44>

$$\int_{x_0}^x F(x) dx = V(x_0) - V(x)$$

<2-45> (2.5.9)

$$V(x) = V(x_0) + \int_{x_0}^x [-F(x)] dx$$

<2-46>

$$\begin{array}{l} V(x_1) - V(x_2) \\ \quad = \int_{x_0}^{x_1} F(x) dx - \int_{x_0}^{x_2} F(x) dx \\ \quad = \int_{x_0}^{x_1} F(x) dx + \int_{x_0}^{x_2} F(x) dx \\ \quad = \int_{x_1}^{x_0} F(x) dx + \int_{x_0}^{x_2} F(x) dx \\ \quad = \int_{x_1}^{x_2} F(x) dx \end{array}$$

<2-47> (2.5.10)

$$V(x) = - \int_{x_0}^x F(x) dx$$

<2-48> (2.5.11)

$$\frac{m}{2} v^2 + V(x) = E$$

<2-49> (2.6.1)

$$v^2 = \frac{2[E - V(x)]}{m}$$

<2-50> (2.6.2)

$$E - V(x) \geq 0$$

<2-51> (2.6.3)

$$\text{When } a \geq x \geq b, \quad v^2 = \frac{2[E - V(x)]}{m} \geq 0$$

<2-52> (2.6.4)

$$v = \frac{dx}{dt} = \pm \sqrt{\frac{2}{m}} \sqrt{E - V(x)}$$

<2-53> (2.6.5)

$$\begin{array}{l} \int \frac{1}{\sqrt{E - V(x)}} dx \\ \quad = \int \sqrt{\frac{2}{m}} \int dt + C \\ \quad = \sqrt{\frac{2}{m}} t + C \end{array}$$

<2-54> (2.6.6)

$$\begin{array}{l} \left\{ \begin{array}{l} x(t) = x_0 + v_0 t \\ v(t) = v_0 \end{array} \right. \end{array}$$

<2-55> (2.6.7)

$$\frac{m}{2} v^2(t) = E$$

<2-56> (2.6.8)

$$E = \frac{1}{2}mv_0^2$$

<2-57> (2.6.9)

$$\int \frac{1}{\sqrt{E}} dx = \pm \sqrt{\frac{2}{m}}t + C$$

<2-58> (2.6.10)

$$\sqrt{\frac{2}{m}} \frac{x}{v_0} = \pm \sqrt{\frac{2}{m}}t + C$$

<2-59> (2.6.11)

$$C = \sqrt{\frac{2}{m}} \frac{x_0}{v_0}$$

<2-60>

$$\sqrt{\frac{2}{m}} \frac{x - x_0}{v_0} = \pm \sqrt{\frac{2}{m}}t$$

<2-61>

$$x - x_0 = \pm \sqrt{\frac{2}{m}}t \sqrt{\frac{m}{2}}v_0 = \pm v_0 t$$

<2-62> (2.6.12)

$$x = x_0 \pm v_0 t$$

<2-63>

$$v(t) = \frac{dx(t)}{dt} = \pm v_0$$

<2-64> (2.6.13)

$$v(t) = v_0$$

<2-65> (2.6.14)

$$\begin{array}{l} \left\{ \begin{array}{l} x(t) = x_0 + v_0 t \\ v(t) = v_0 \end{array} \right. \end{array}$$

<2-66> (2.7.1)

$$m \frac{d^2x}{dt^2} = \begin{array}{l} -R(v), \quad \text{when } v \geq 0 \text{ travelling to the right} \\ +R(v), \quad \text{when } v \leq 0 \text{ travelling to the left} \end{array}$$

<2-67> (2.7.2)

$$m \frac{dv}{dt} = -R(v)$$

<2-68> (2.7.3)

$$\begin{array}{l} \int R(v) dv = -\int dt + C \\ \int -t + C \\ \end{array}$$

<2-69> (2.7.4)

$$R(v) = \begin{array}{l} \alpha v \text{ (for small } v) \\ \beta v^2 \text{ (for large } v) \end{array}$$

<2-70>

$$\begin{array}{l} \int \frac{1}{\alpha v} dv = \frac{1}{\alpha} \ln v \\ = -t + C \text{ (for small } v) \\ \int \frac{1}{\beta v^2} dv = -\frac{1}{\beta v} = -t + C \text{ (for large } v) \end{array}$$

<2-71>

$$C = \begin{array}{l} \frac{1}{\alpha} \ln v_0 \text{ (for small } v) \\ -\frac{1}{\beta v_0} \text{ (for large } v) \end{array}$$

<2-72> (2.7.5)

$$v(t) = \begin{array}{l} v_0 e^{-\alpha t} \text{ (for small } v) \\ -\frac{v_0}{1 + (\beta/m)v_0 t} \text{ (for large } v) \end{array}$$

<2-73> (2.7.6)

$$x(t) = \begin{array}{l} v_0 \int e^{-(\alpha/m)t} dt + C' = \frac{mv_0}{\alpha} e^{-(\alpha/m)t} + C' \\ \text{(for small } v) \\ v_0 \int \frac{1}{1 + (\beta/m)v_0 t} dt + C' \\ = \frac{\beta}{m} \ln \left(1 + \frac{v_0 \beta}{m} t \right) + C' \text{ (for large } v) \end{array}$$

$\end{array}\right.$

<2-74> (2.7.7)

$C' = \left\{ \begin{array}{l} x_0 + \frac{mv_0}{\alpha} \quad \& \quad \text{for small } v \\ x_0 \quad \& \quad \text{for large } v \end{array} \right.$

<2-75> (2.7.8)

$x(t) = \left\{ \begin{array}{l} x_0 + \frac{mv_0}{\alpha} \left[1 - e^{-(\alpha/m)t} \right] \quad \& \quad \text{for small } v \\ x_0 + \frac{v_0}{\beta} \ln \left(1 + \frac{v_0 \beta}{m} t \right) \quad \& \quad \text{for large } v \end{array} \right.$

<2-76>

$x(t) \rightarrow \left\{ \begin{array}{l} x_0 + \frac{mv_0}{\alpha} \quad \& \quad \text{for small } v \\ \infty \quad \& \quad \text{for large } v \end{array} \right.$

<3-1> (3.1.1)

$E = \frac{1}{2}mv^2 + V(x)$

<3-2> (3.1.2)

$F(x) = -\frac{dV(x)}{dx}$

<3-3> (3.1.3)

$F(x) = -kx$

<3-4> (3.1.4)

$m \frac{d^2x}{dt^2} = -kx$

<3-5> (3.1.5)

$\frac{d^2x}{dt^2} = -\omega^2 x$

<3-6> (3.1.6)

$x(t) = A \sin(\omega t + B)$

<3-7> (3.1.7)

$\frac{dx}{dt} = A \omega \cos(\omega t + B)$

<3-8> (3.1.8)

$$\left\{ \begin{array}{l} a = A \sin B \\ \dot{a} = A \omega \cos B \end{array} \right.$$

<3-9> (3.1.9)

$$A = a, \quad B = \frac{\pi}{2}$$

<3-10> (3.1.10)

$$\left\{ \begin{array}{l} x(t) = a \sin\left(\omega t + \frac{\pi}{2}\right) = a \cos(\omega t) \\ v(t) = a \omega \cos\left(\omega t + \frac{\pi}{2}\right) = -a \omega \sin(\omega t) \end{array} \right.$$

<3-11> (3.1.11)

$$V(x) = -\int F(x) dx$$

<3-12> (3.1.12)

$$V(x) = -\int (-kx) dx = \frac{1}{2} kx^2$$

<3-13> (3.1.13)

$$F(x_0) = -V^{(1)}(x_0) \equiv V'(x_0) = 0$$

<3-14> (3.1.14)

$$\begin{array}{l} V(x) = V(x_0) + \frac{V^{(1)}(x_0)}{1!} (x-x_0) + \\ \quad + \frac{V^{(2)}(x_0)}{2!} (x-x_0)^2 + \frac{V^{(3)}(x_0)}{3!} (x-x_0)^3 + \dots \end{array}$$

<3-15> (3.1.15)

$$V(x) = V(x_0) + \frac{V^{(2)}(x_0)}{2!} (x-x_0)^2$$

<3-16> (3.1.16)

$$V^{(2)}(x_0) \equiv k \quad \text{\$k\$ is a constant.}$$

<3-17> (3.1.17)

$$V(x) = V(0) + \frac{1}{2} kx^2$$

<3-18> (3.2.1)

$$m \frac{d^2x}{dt^2} = -kx - \alpha \frac{dx}{dt}$$

<3-19> (3.2.2)

$$\left\{ \begin{array}{l} \sqrt{\frac{k}{m}} = \omega \\ \frac{\alpha}{m} = 2\gamma \end{array} \right.$$

<3-20> (3.2.3)

$$\frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega^2 x = 0$$

<3-21> (3.2.4)

$$x(t) = C_1 e^{\left(-\gamma + \sqrt{\gamma^2 - \omega^2}\right)t} + C_2 e^{\left(-\gamma - \sqrt{\gamma^2 - \omega^2}\right)t}$$

<3-22> (3.2.5)

$$\left\{ \begin{array}{l} \frac{dx(t)}{dt} = \left(-\gamma + \sqrt{\gamma^2 - \omega^2}\right) C_1 e^{\left(-\gamma + \sqrt{\gamma^2 - \omega^2}\right)t} \\ \quad + \left(-\gamma - \sqrt{\gamma^2 - \omega^2}\right) C_2 e^{\left(-\gamma - \sqrt{\gamma^2 - \omega^2}\right)t} \end{array} \right.$$

<3-23> (3.2.6)

$$\left\{ \begin{array}{l} C_1 + C_2 = a \\ \left(-\gamma + \sqrt{\gamma^2 - \omega^2}\right) C_1 + \left(-\gamma - \sqrt{\gamma^2 - \omega^2}\right) C_2 = 0 \end{array} \right.$$

<3-24> (3.2.7)

$$\left\{ \begin{array}{l} C_1 = \frac{1}{2} \left(1 - \frac{\gamma}{\sqrt{\gamma^2 - \omega^2}}\right) a \\ C_2 = \frac{1}{2} \left(1 + \frac{\gamma}{\sqrt{\gamma^2 - \omega^2}}\right) a \end{array} \right.$$

<3-25> (3.2.8)

$$\begin{array}{l} x(t) = \frac{a}{2} e^{\left(1 - \sqrt{\gamma^2 - \omega^2}\right)t} \cos\left(\sqrt{\gamma^2 - \omega^2}t\right) \\ + \frac{a}{2} e^{\left(1 + \sqrt{\gamma^2 - \omega^2}\right)t} \cos\left(\sqrt{\gamma^2 - \omega^2}t\right) \end{array}$$

<3-26> (3.2.9)

$$\begin{array}{l} x(t) = \frac{a}{2} e^{\left(-\gamma + \sqrt{\gamma^2 - \omega^2}\right)t} \cos\left(\sqrt{\gamma^2 - \omega^2}t\right) \\ + \frac{a}{2} e^{\left(-\gamma - \sqrt{\gamma^2 - \omega^2}\right)t} \cos\left(\sqrt{\gamma^2 - \omega^2}t\right) \end{array}$$

<3-27> (3.2.10)

$$\begin{array}{l} x(t) = \frac{a}{2} e^{-\gamma t} \cos\left(\sqrt{\omega^2 - \gamma^2}t\right) \\ + \frac{a}{2} e^{-\gamma t} \sin\left(\sqrt{\omega^2 - \gamma^2}t\right) \end{array}$$

<3-28> (3.2.11)

$$\begin{array}{l} e^{i\sqrt{\omega^2 - \gamma^2}t} = \cos\left(\sqrt{\omega^2 - \gamma^2}t\right) \\ + i \sin\left(\sqrt{\omega^2 - \gamma^2}t\right) \end{array}$$

<3-29> (3.2.12)

$$\begin{array}{l} x(t) = a e^{-\gamma t} \cos\left(\sqrt{\omega^2 - \gamma^2}t\right) \\ + \frac{a}{\sqrt{\omega^2 - \gamma^2}} e^{-\gamma t} \sin\left(\sqrt{\omega^2 - \gamma^2}t\right) \end{array}$$

<3-30> (3.3.1)

$$m \frac{d^2x}{dt^2} = -kx + F \cos(\Omega t)$$

<3-31> (3.3.2)

$$\frac{d^2x}{dt^2} + \omega^2 x = f \cos(\Omega t)$$

<3-32> (3.3.3)

$$x(t) = C_1 e^{i\omega t} + C_2 e^{-i\omega t} + \frac{f}{\left| \omega^2 - \Omega^2 \right|} \cos(\Omega t)$$

<3-33> (3.3.4)

$$\begin{array}{l} \frac{dx}{dt} = i\omega \left(C_1 e^{i\omega t} - C_2 e^{-i\omega t} \right) \\ \frac{f}{\left| \omega^2 - \Omega^2 \right|} \sin(\Omega t) \end{array}$$

<3-34> (3.3.5)

$$i\omega(C_1 - C_2) = 0$$

<3-35> (3.3.6)

$$x(t) = 2C_1 \cos(\omega t) + \frac{f}{\left| \omega^2 - \Omega^2 \right|} \cos(\Omega t)$$

<3-36>

$$C_1 = \frac{1}{2} \left(a - \frac{f}{\left| \omega^2 - \Omega^2 \right|} \right)$$

<3-37>

$$\cos A - \cos B = -2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

<3-38> (3.3.7)

$$\begin{array}{l} x(t) = a \cos(\omega t) + \frac{2f}{\left| \omega^2 - \Omega^2 \right|} \sin \left[\frac{\omega + \Omega}{2} t \right] \sin \left[\frac{\omega - \Omega}{2} t \right] \end{array}$$

<3-39> (3.4.1)

$$m \frac{d^2x}{dt^2} = -kx - \alpha \frac{dx}{dt} + F \cos(\Omega t)$$

<3-40> (3.4.2)

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \omega^2 x = f \cos(\Omega t)$$

<3-41> (3.4.3)

$$\begin{array}{l} x(t) = e^{-\gamma t} \left[C_1 e^{\sqrt{\gamma^2 - \omega^2} t} + C_2 e^{-\sqrt{\gamma^2 - \omega^2} t} \right] + \frac{f \cos(\Omega t - \phi)}{\omega^2 - \gamma^2} \end{array}$$

<3-42> (3.4.4)

$$\tan \phi = \frac{2\gamma\Omega}{\omega^2 - \Omega^2}$$

<3-43>

$$m v^2 + V(x) = \frac{m}{2} a^2 \omega^2 \equiv E$$

<4-1> (4.1.1)

$$\begin{equation*} \vec{r} = \vec{i}x + \vec{j}y \\ \end{equation*}$$

<4-2> (4.1.2)

$$\vec{r} = \vec{i}x + \vec{j}y + \vec{k}z$$

<4-3> (4.1.3)

$$\vec{r} = \vec{i}x + \vec{j}y$$

<4-4> (4.1.4)

$$\left\{ \begin{array}{l} \\ & x = r \cos \theta \\ & y = r \sin \theta \end{array} \right. \\ \text{right.}$$

<4-5> (4.1.5)

$$\left\{ \begin{array}{l} \\ r = \sqrt{x^2 + y^2} \\ \theta = \tan^{-1} \left(\frac{y}{x} \right) \end{array} \right. \\ \text{right.}$$

<4-6> (4.1.6)

$$\rho = r \cos \left(\frac{\pi}{2} - \theta \right) = r \sin \theta$$

<4-7> (4.1.7)

$$\left\{ \begin{array}{l} \\ x = \rho \cos \phi \\ y = \rho \sin \phi \end{array} \right. \\ \text{right.}$$

<4-8> (4.1.8)

$$\begin{aligned} & \left\{ \begin{aligned} x &= r \sin \theta \cos \phi \\ y &= r \sin \theta \sin \phi \end{aligned} \right. \end{aligned}$$

<4-9> (4.1.9)

$$z = r \cos \theta$$

<4-10> (4.1.10)

$$\begin{aligned} & \left\{ \begin{aligned} x &= r \sin \theta \cos \phi \\ y &= r \sin \theta \sin \phi \\ z &= r \cos \theta \end{aligned} \right. \\ & \text{right., quad} \\ & (-\pi \leq \theta < \pi, 0 \leq \phi < 2\pi) \end{aligned}$$

<4-11> (4.1.11)

$$\begin{aligned} & \left\{ \begin{aligned} r &= \sqrt{x^2 + y^2 + z^2} \\ \theta &= \tan^{-1} \left(\frac{z}{\sqrt{x^2 + y^2}} \right) \\ \phi &= \tan^{-1} \left(\frac{y}{x} \right) \end{aligned} \right. \\ & \text{right.} \end{aligned}$$

<4-12>

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

<4-13> (4.2.1)

$$\begin{aligned} & \left. \begin{aligned} \frac{d^2 x(t)}{dt^2} &= F_x \\ \frac{d^2 y(t)}{dt^2} &= F_y \\ \frac{d^2 z(t)}{dt^2} &= F_z \end{aligned} \right\} \text{quad} \rightarrow \text{quad} \\ & \frac{d^2 \vec{r}(t)}{dt^2} = \vec{F} \end{aligned}$$

<4-14> (4.2.2)

$$\left\{ \begin{aligned} \end{aligned} \right.$$

$\vec{r}(0) \equiv \vec{r}_0 = v_0 \hat{i} + h \hat{k}$
 $\vec{v}(0) \equiv \vec{v}_0 = v_0 \cos \theta \hat{i} + v_0 \sin \theta \hat{j} + \hat{k}$

<4-15>

$$\vec{r}(t) = x(t) \hat{i} + y(t) \hat{j} + z(t) \hat{k}$$

<4-16> (4.2.3)

$$m \frac{d^2 \vec{r}(t)}{dt^2} = \vec{F}$$

<4-17> (4.2.4)

$$\vec{F} = -m \hat{j} g$$

<4-18> (4.2.5)

$$\begin{array}{l} \left\{ \begin{array}{l} \frac{d^2 x(t)}{dt^2} = 0 \\ \frac{d^2 y(t)}{dt^2} = -mg \\ \frac{d^2 z(t)}{dt^2} = 0 \end{array} \right. \end{array}$$

<4-19> (4.2.6)

$$\begin{array}{l} \left\{ \begin{array}{l} \left\{ \begin{array}{l} x(t) = (v_0 \cos \theta) t \\ v_x(t) = v_0 \cos \theta \end{array} \right. \\ \left\{ \begin{array}{l} y(t) = h + (v_0 \sin \theta) t - \frac{1}{2} g t^2 \\ v_y(t) = v_0 \sin \theta - g t \end{array} \right. \\ \left\{ \begin{array}{l} z(t) = 0 \\ v_z(t) = 0 \end{array} \right. \end{array} \right. \end{array}$$

<4-20> (4.2.7)

$$\begin{array}{l} \left\{ \begin{array}{l} \left\{ \begin{array}{l} x(t) = (v_0 \cos \theta) t \\ y(t) = h + (v_0 \sin \theta) t - \frac{1}{2} g t^2 \end{array} \right. \end{array} \right. \end{array}$$

<4-21>

$$t = \frac{v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gh}}{g}$$

<4.22> (4.2.8)

$$y = -\frac{g}{2v_0^2 \cos^2 \theta} (x-X)^2 + \frac{v_0^2 \sin^2 \theta}{2g}$$

<4-23>

$$X = \frac{v_0^2 \sin(2\theta)}{2g}$$

<4-24>

$$y = \frac{v_0^2 \sin^2 \theta}{2g}$$

<4-25>

$$\left(\frac{dr}{dt}, \frac{d\theta}{dt} \right) \equiv \omega$$

<4-26> (4.3.1)

$$\left(\begin{array}{l} \frac{dx}{dt} = \frac{dr}{dt} \cos \theta - r \omega \sin \theta \\ \frac{dy}{dt} = \frac{dr}{dt} \sin \theta + r \omega \cos \theta \end{array} \right)$$

<4-27>

$$\frac{d(f(x(t)))}{dt} = \frac{df(x)}{dx} \frac{dx(t)}{dt}$$

<4-28>

$$\frac{dx}{dt} \cos \theta + \frac{dy}{dt} \sin \theta = \frac{dr}{dt}$$

<4-29>

$$-\frac{dx}{dt} \sin \theta + \frac{dy}{dt} \cos \theta = r \omega$$

<4-30> (4.3.2)

$$\left(\begin{array}{l} \frac{dr}{dt} = \frac{dx}{dt} \cos \theta + \frac{dy}{dt} \sin \theta \\ \frac{d\theta}{dt} = -\frac{dx}{dt} \sin \theta + \frac{dy}{dt} \cos \theta \end{array} \right)$$

<4-31> (4.3.3)

$$\left(\begin{array}{l} \frac{d^2x}{dt^2} = \left(\frac{d^2r}{dt^2} - r \omega^2 \right) \cos \theta - \left(r \frac{d\omega}{dt} + 2 \frac{dr}{dt} \omega \right) \sin \theta \\ \frac{d^2y}{dt^2} = \left(\frac{d^2r}{dt^2} - r \omega^2 \right) \sin \theta + \left(r \frac{d\omega}{dt} + 2 \frac{dr}{dt} \omega \right) \cos \theta \end{array} \right)$$

$\end{array}\right.$

<4-32>

$$\frac{d^2x}{dt^2}\cos\theta + \frac{d^2y}{dt^2}\sin\theta = \frac{d^2r}{dt^2} - r\omega^2$$

<4-33>

$$-\frac{d^2x}{dt^2}\sin\theta + \frac{d^2y}{dt^2}\cos\theta = r\frac{d\omega}{dt} + 2\frac{dr}{dt}\omega$$

<4-34>

$$\frac{d\{f(t)g(t)\}}{dt} = \frac{df(t)}{dt}g(t) + f(t)\frac{dg(t)}{dt}$$

<4-35>

$$\frac{d(r^2\omega)}{dt} = 2r\frac{dr}{dt}\omega + r^2\frac{d\omega}{dt}$$

<4-36>

$$\left\{ \begin{array}{l} \frac{d^2r}{dt^2} - r\omega^2 = \frac{d^2x}{dt^2}\cos\theta + \frac{d^2y}{dt^2}\sin\theta \\ \frac{1}{r}\frac{d(r^2\omega)}{dt} = -\frac{d^2x}{dt^2}\sin\theta + \frac{d^2y}{dt^2}\cos\theta \end{array} \right.$$

<4-37> (4.3.5)

$$\left\{ \begin{array}{l} \frac{d^2x}{dt^2} = F_x \\ \frac{d^2y}{dt^2} = F_y \end{array} \right.$$

<4-38> (4.3.6)

$$\left\{ \begin{array}{l} \left(\frac{d^2r}{dt^2} - r\omega^2 \right) = F_x\cos\theta + F_y\sin\theta \\ \frac{1}{r}\frac{d(r^2\omega)}{dt} = -F_x\sin\theta + F_y\cos\theta \end{array} \right.$$

<4-39> (4.3.7)

$$\vec{F} = \frac{d\vec{r}}{dt}$$

<4-40> (4.3.8)

$$\vec{F} = -\frac{\vec{r}}{r} F$$

<4-41>

$$\frac{\vec{r}}{r} = \vec{i} \cos \theta + \vec{j} \sin \theta$$

<4-42>

$$\vec{F} = -\vec{i} F \cos \theta - \vec{j} F \sin \theta$$

<4-43> (4.3.9)

$$\begin{array}{l} \left\{ \begin{array}{l} F_x = -F \cos \theta \\ F_y = -F \sin \theta \end{array} \right. \end{array}$$

<4-44> (4.3.10)

$$\begin{array}{l} \left\{ \begin{array}{l} \displaystyle m \left(\frac{d^2 r}{dt^2} - r \omega^2 \right) = -F \\ \displaystyle \frac{m}{r} \frac{d(r^2 \omega)}{dt} = 0 \end{array} \right. \end{array}$$

<4-45> (4.3.11)

$$m \frac{d^2 r}{dt^2} = -F + \frac{m C^2}{r^3}$$

<4-46> (4.3.12)

$$F = \frac{m C^2}{\ell^3}$$

<4-47>

$$r^2 \omega = \ell^2 \Omega = C$$

<4-48> (4.3.13)

$$\omega = \frac{C}{\ell^2} = \text{constant} \equiv \omega_0$$

<4-49> (4.4.1)

$$\vec{F} = \vec{j} mg$$

<4-50> (4.4.2)

$$\vec{S} = -\frac{\vec{r}}{r} S$$

<4-51> (4.4.3)

$$m \frac{d^2 \vec{r}}{dt^2} = \vec{F} + \vec{S}$$

<4.52> (4.4.4)

$$\left\{ \begin{array}{l} \frac{d^2x}{dt^2} = -\frac{x}{r}S \\ \frac{d^2y}{dt^2} = mg - \frac{y}{r}S \\ \frac{d^2z}{dt^2} = 0 \end{array} \right.$$

<4.53> (4.4.5)

$$\left\{ \begin{array}{l} x = \ell \sin \phi \\ y = \ell \cos \phi \end{array} \right.$$

<4-54> (4.4.6)

$$\left\{ \begin{array}{l} x = \ell \phi \\ y = \ell \end{array} \right.$$

<4-55> (4.4.7)

$$\left\{ \begin{array}{l} \frac{d^2x}{dt^2} = -\frac{x}{\ell}S \\ 0 = mg - S \end{array} \right.$$

<4-56>

$$\frac{d^2x}{dt^2} = -\frac{g}{\ell}x$$

<4-57> (4.4.8)

$$\omega = \sqrt{\frac{g}{\ell}}$$

<4-58> (4.4.9)

$$\frac{d^2x}{dt^2} = -\omega^2x$$

<4-59> (4.4.10)

$$x(t) = A \sin(\omega t + B)$$

<4-60> (4.4.11)

$$x(t) = a \cos(\omega t)$$

<4-61> (4.4.12)

$$\begin{array}{l} \left\{ \begin{array}{l} \omega = \sqrt{\frac{g}{\ell}} \\ T = 2\pi \sqrt{\frac{\ell}{g}} \end{array} \right. \end{array}$$

<4-62> (4.4.13)

$$\begin{array}{l} m \frac{d^2 \vec{r}}{dt^2} = -k \vec{r} \\ \frac{d^2 x}{dt^2} = -kx \\ \frac{d^2 y}{dt^2} = -ky \end{array}$$

<4-63> (4.4.14)

$$\begin{array}{l} \left\{ \begin{array}{l} x(t) = a_1 \sin(\omega t + b_1) \\ y(t) = a_2 \sin(\omega t + b_2) \end{array} \right. \end{array}$$

<4-64>

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

<4-65>

$$\sin \omega t = \frac{1}{\sin(b_2 - b_1)} \left(\frac{x}{a_1} \sin b_2 - \frac{y}{a_2} \sin b_1 \right)$$

<4-66>

$$\cos \omega t = \frac{1}{\sin(b_2 - b_1)} \left(-\frac{x}{a_1} \cos b_2 + \frac{y}{a_2} \cos b_1 \right)$$

<4-67>

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

<4-68> (4.4.15)

$$\sin^2(b_2 - b_1) = \frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} - 2 \frac{x}{a_1} \frac{y}{a_2} \cos(b_2 - b_1)$$

<4-69> (4.4.16)

$$1 = \frac{x^2}{a_1^2} + \frac{y^2}{a_2^2}$$

<4-70> (4.5.1)

$$m \frac{d^2 \vec{r}(t)}{dt^2} = \vec{F}(x, y, z)$$

<4-71> (4.5.2)

$$m \frac{d\vec{v}}{dt} = \vec{F}(x,y,z)$$

<4-72> (4.5.3)

$$m \left(\vec{v} \cdot \frac{d\vec{v}}{dt} \right) = \vec{v} \cdot \vec{F}(x,y,z)$$

<4-73> (4.5.4)

$$\vec{v} \cdot \vec{v} = v_x^2 + v_y^2 + v_z^2 \equiv v^2$$

<4-74> (4.5.5)

$$\begin{array}{l} \frac{d(v^2)}{dt} \\ = \frac{d(v_x^2 + v_y^2 + v_z^2)}{dt} \\ = 2 \left(v_x \frac{dv_x}{dt} + v_y \frac{dv_y}{dt} + v_z \frac{dv_z}{dt} \right) \\ = 2 \left(\vec{v} \cdot \frac{d\vec{v}}{dt} \right) \end{array}$$

<4-75> (4.5.6)

$$\frac{m}{2} \frac{d(v^2)}{dt} = \frac{d \left(m \frac{v^2}{2} \right)}{dt} = \vec{v} \cdot \vec{F}(x,y,z)$$

<4-76> (4.5.7)

$$\int_{t_1}^{t_2} \frac{dT}{dt} dt = \int_{t_1}^{t_2} \left(\vec{v} \cdot \vec{F}(x,y,z) \right) dt$$

<4-77> (4.5.8)

$$\int_{T_1}^{T_2} dT = T_2 - T_1$$

<4-78> (4.5.9)

$$\begin{array}{l} \int_{t_1}^{t_2} \vec{v} \cdot \vec{F}(x,y,z) dt \\ = \int_{t_1}^{t_2} \left(F_x \frac{dx}{dt} + F_y \frac{dy}{dt} + F_z \frac{dz}{dt} \right) dt \\ = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz \\ \equiv \int_{P_1}^{P_2} \left(F_x dx + F_y dy + F_z dz \right) \end{array}$$

<4-79> (4.5.10)

$$T_2 - T_1 = \int_{P_1}^{P_2} \left(F_x dx + F_y dy + F_z dz \right)$$

<4-80> (4.5.11)

$$\begin{array}{l} F_x \equiv \frac{\partial V(x,y,z)}{\partial x} \\ F_y \equiv \frac{\partial V(x,y,z)}{\partial y} \end{array}$$

$$\vec{F} = -\text{grad} V(x,y,z)$$

<4-81> (4.5.12)

$$\int_{P_1}^{P_2} \vec{F} \cdot d\vec{S} = -\int_{P_1}^{P_2} \text{grad} V(x,y,z) \cdot d\vec{S}$$

<4-82>

$$d\vec{S} = \vec{i} dx + \vec{j} dy + \vec{k} dz$$

<4-83> (4.5.13)

$$\text{grad} V(x,y,z) \cdot d\vec{S} = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

<4-84> (4.5.14)

$$dV(x,y,z) = V(x+dx, y+dy, z+dz) - V(x,y,z) = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

<4-85> (4.5.15)

$$\int_{P_1}^{P_2} \vec{F} \cdot d\vec{S} = -\int_{P_1}^{P_2} dV = V_1 - V_2$$

<4-86>

$$T_2 - T_1 = V_1 - V_2$$

<4-87> (4.5.16)

$$T_1 + V_1 = T_2 + V_2$$

<4-88> (4.6.1)

$$\vec{L}(t) = \vec{r}(t) \times \vec{p}(t)$$

<4-89> (4.6.2)

$$|\vec{L}| = |\vec{r}| |\vec{p}| \sin \theta$$

<4-90> (4.6.3)

$$\vec{r}(t) = \vec{i}x + \vec{j}y + \vec{k}z$$

<4-91> (4.6.4)

$$\vec{F}(\vec{r}) = \vec{e}_r f(r)$$

<4-92> (4.6.5)

$$\vec{e}_r = \frac{1}{r} \vec{r} = \frac{\vec{r}}{r}$$

<4-93>

$$\vec{e}_r \cdot \vec{e}_r = 1$$

<4-94> (4.6.6)

$$\begin{aligned} x &= r \sin \theta \cos \phi \\ y &= r \sin \theta \sin \phi \\ z &= r \cos \theta \end{aligned}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \left(\frac{\sqrt{x^2 + y^2}}{z} \right)$$

$$\phi = \tan^{-1} \left(\frac{y}{x} \right)$$

<4-95> (4.6.7)

$$\vec{r} \times \vec{F}(\vec{r}) = 0$$

<4-96> (4.6.8)

$$m \frac{d^2 \vec{r}}{dt^2} = \vec{F}$$

<4-97> (4.6.9)

$$m \frac{d \vec{v}}{dt} = \vec{F}$$

<4-98> (4.6.10)

$$m \left(\vec{r} \times \frac{d \vec{v}}{dt} \right)$$

$$= \vec{r} \times \vec{F} = 0$$

<4-99>

$$\begin{aligned} &\frac{d}{dt} \left(\vec{r} \times \vec{v} \right) \\ &= \frac{d \vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d \vec{v}}{dt} \\ &= \vec{v} \times \vec{v} + \vec{r} \times \frac{d \vec{v}}{dt} \\ &= \vec{r} \times \frac{d \vec{v}}{dt} \end{aligned}$$

<4-100> (4.6.11)

$$\begin{aligned} &\frac{d}{dt} \left(\vec{r} \times \frac{d \vec{v}}{dt} \right) \\ &= \frac{d}{dt} \left(\vec{r} \times \vec{p} \right) \\ &= \frac{d \vec{L}}{dt} \\ &= 0 \end{aligned}$$

<4-101> (4.6.12)

$$m \frac{d^2 \vec{r}}{dt^2} = \vec{F} \quad \rightarrow \quad \frac{d \vec{L}}{dt} = 0$$

$$\begin{array}{l} \\ \displaystyle{m\frac{d^2x}{dt^2}}=F_x \\ \displaystyle{m\frac{d^2y}{dt^2}}=F_y \\ \end{array}$$

<4-102> (4.6.13)

$$\left\{ \begin{array}{l} \\ \displaystyle{\frac{dx}{dt}=\frac{dr}{dt}\cos\theta-r\frac{d\theta}{dt}\sin\theta} \\ \displaystyle{\frac{dy}{dt}=\frac{dr}{dt}\sin\theta+r\frac{d\theta}{dt}\cos\theta} \\ \end{array} \right.$$

<4-103> (4.6.14)

$$\left\{ \begin{array}{l} \\ \displaystyle{\frac{d^2x}{dt^2}=\left(\frac{d^2r}{dt^2}-r\omega^2\right)\cos\theta-2\frac{dr}{dt}\omega+r\frac{d\omega}{dt}\sin\theta} \\ \displaystyle{\frac{d^2y}{dt^2}=\left(\frac{d^2r}{dt^2}-r\omega^2\right)\sin\theta+2\frac{dr}{dt}\omega+r\frac{d\omega}{dt}\cos\theta} \\ \end{array} \right.$$

<4-104>

$$\left\{ \begin{array}{l} \\ \displaystyle{m\left(\frac{d^2r}{dt^2}-r\omega^2\right)\cos\theta-m\left(2\frac{dr}{dt}\omega+r\frac{d\omega}{dt}\sin\theta\right)=F_x} \\ \displaystyle{m\left(\frac{d^2r}{dt^2}-r\omega^2\right)\sin\theta+m\left(2\frac{dr}{dt}\omega+r\frac{d\omega}{dt}\cos\theta\right)=F_y} \\ \end{array} \right.$$

<4-105> (4.6.15)

$$\left\{ \begin{array}{l} \\ \displaystyle{m\left(\frac{d^2r}{dt^2}-r\omega^2\right)} \\ =F_x\cos\theta+F_y\sin\theta \\ \displaystyle{m\left(2\frac{dr}{dt}\omega+r\frac{d\omega}{dt}\right)} \\ =-F_x\sin\theta+F_y\cos\theta \\ \end{array} \right.$$

<4-106> (4.6.16)

$$\left\{ \begin{array}{l} \\ \displaystyle{F_x\cos\theta+F_y\sin\theta=F_r} \\ \displaystyle{-F_x\sin\theta+F_y\cos\theta=F_\theta} \\ \end{array} \right.$$

<4-107> (4.6.17)

$$m\left(\frac{d^2r}{dt^2} - r\omega^2\right) = F_r$$

<4-108> (4.6.18)

$$m\left(2\frac{dr}{dt}\omega + r\frac{d\omega}{dt}\right) = F_\theta$$

<4-109> (4.6.19)

$$\begin{array}{l} \left\{ \begin{array}{l} F_x = \frac{x}{r}f(r) \cos\theta \\ F_y = \frac{y}{r}f(r) \sin\theta \end{array} \right. \\ \end{array}$$

<4-110> (4.6.20)

$$\begin{array}{l} \left\{ \begin{array}{l} F_r = F_x \cos\theta + F_y \sin\theta \\ \quad = f(r) \cos^2\theta + f(r) \sin^2\theta \\ \quad = f(r) \\ F_\theta = -F_x \sin\theta + F_y \cos\theta \\ \quad = -f(r) \cos\theta \sin\theta + f(r) \sin\theta \cos\theta \\ \quad = 0 \end{array} \right. \\ \end{array}$$

<4-111> (4.6.21)

$$m\left(\frac{d^2r}{dt^2} - r\omega^2\right) = f(r)$$

<4-112> (4.6.22)

$$m\left(2\frac{dr}{dt}\omega + r\frac{d\omega}{dt}\right) = 0$$

<4-113>

$$\frac{d}{dt}(f(t)g(t)) = \frac{df(t)}{dt}g(t) + f(t)\frac{dg(t)}{dt}$$

<4-114>

$$\begin{array}{l} \left\{ \begin{array}{l} \frac{d(r^2\omega)}{dt} = 2r\frac{dr}{dt}\omega + r^2\frac{d\omega}{dt} \\ \quad = r\left(2\frac{dr}{dt}\omega + r\frac{d\omega}{dt}\right) \end{array} \right. \\ \end{array}$$

<4-115> (4.6.23)

$$\frac{m}{r}\frac{d(r^2\omega)}{dt} = 0$$

<4-116> (4.6.24)

$$mr^2\omega = h$$

<4-117>

$$\vec{a} \times \vec{b} = \vec{i} (a_y b_z - a_z b_y) + \vec{j} (a_z b_x - a_x b_z) + \vec{k} (a_x b_y - a_y b_x)$$

<4-118> (4.6.25)

$$L_z = x p_y - y p_x = m \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right)$$

<4-119> (4.6.26)

$$\begin{array}{l} \begin{array}{l} L_z = m r^2 \dot{\theta} \left(\dot{\phi} \sin \theta + r \dot{\theta} \cos \theta \right) \\ - m r^2 \dot{\theta} \left(\dot{\phi} \cos \theta - r \dot{\theta} \sin \theta \right) \end{array} \\ = m r^2 \dot{\theta} \dot{\phi} \end{array}$$

<4-120>

$$\begin{array}{l} \frac{dA}{dt} \\ = \frac{d}{dt} \left(r^2 \dot{\theta} \right) \\ = 2 r \dot{r} \dot{\theta} + r^2 \ddot{\theta} \\ = r^2 \dot{\theta} \omega \end{array}$$

<4-121> (4.6.27)

$$L_z = 2m \frac{dA}{dt}$$

<4-122> (4.7.1)

$$\vec{F} = \frac{\vec{r}}{r} \frac{K}{r^2}$$

<4-123> (4.7.2)

$$\begin{array}{l} \left[\begin{array}{l} F_x = K \frac{\cos \theta}{r^2} \\ F_y = K \frac{\sin \theta}{r^2} \end{array} \right] \end{array}$$

<4-124> (4.7.3)

$$\begin{array}{l} \begin{array}{l} F_r = F_x \cos \theta + F_y \sin \theta \\ = K \frac{\cos^2 \theta}{r^2} + K \frac{\sin^2 \theta}{r^2} \\ = K \frac{1}{r^2} \\ F_{\theta} = -F_x \sin \theta + F_y \cos \theta \\ = -K \frac{\cos \theta}{r^2} \sin \theta + K \frac{\sin \theta}{r^2} \cos \theta \\ = 0 \end{array} \end{array}$$

<4-125> (4.7.4)

$$m \frac{d}{dt} \left(\frac{d^2 r}{dt^2} - r \omega^2 \right) = \frac{K}{r^2}$$

<4-126> (4.7.5)

$$m \left(2 \frac{dr}{dt} \omega + r \frac{d\omega}{dt} \right) = 0$$

<4-127> (4.7.6)

$$m \frac{dv}{dt} - \frac{h^2}{mr^3} - \frac{K}{r^2} = 0$$

<4-128> (4.7.7)

$$m \frac{dv}{dt} = \frac{h^2}{mr^3} + \frac{K}{r^2}$$

<4-129> (4.7.8)

$$m v \frac{dv}{dt} - \frac{h^2}{mr^3} v - \frac{K}{r^2} v = 0$$

<4-130>

$$\frac{d}{dt} \left(\frac{m}{2} v^2 \right) = m v \frac{dv}{dt}$$

<4-131>

$$\begin{array}{l} \frac{d}{dt} \left(\frac{d}{dt} \left(\frac{h^2}{2mr^2} \right) \right) \\ = \frac{d}{dr} \left(\frac{d}{dt} \left(\frac{h^2}{2mr^2} \right) \right) \frac{dr}{dt} \\ = - \frac{h^2}{mr^3} v \\ \end{array}$$

<4-132>

$$\begin{array}{l} \frac{d}{dt} \left(\frac{d}{dt} \left(\frac{K}{r} \right) \right) \\ = \frac{d}{dr} \left(\frac{d}{dt} \left(\frac{K}{r} \right) \right) \frac{dr}{dt} \\ = \frac{K}{r^2} v \\ \end{array}$$

<4-133> (4.7.9)

$$\frac{d}{dt} \left(\frac{m}{2} v^2 + \frac{h^2}{2mr^2} + \frac{K}{r} \right) = 0$$

<4-134> (4.7.10)

$$\frac{m}{2} v^2 + \frac{h^2}{2mr^2} + \frac{K}{r} = E$$

<4-135> (4.7.11)

$$\frac{K}{r} = V(r)$$

<4-136>

$$\nabla V(r) = \vec{i} \frac{\partial V(r)}{\partial x}$$

$$+\vec{j}\frac{\partial V(r)}{\partial y}$$

$$+\vec{k}\frac{\partial V(r)}{\partial z}$$

<4-137>

$$\begin{array}{l} \frac{\partial V(r)}{\partial x} \\ =\frac{dV(r)}{dr}\frac{\partial r}{\partial x} \\ =\frac{K}{r^2}\frac{2x}{2\sqrt{x^2+y^2}} \\ =\frac{K}{r^2}\frac{x}{r} \end{array}$$

<4-138>

$$\frac{\partial V(r)}{\partial x}=-\frac{K}{r^2}\frac{x}{r}$$

<4-139>

$$\nabla V(r)=-\left(\vec{i}\frac{x}{r}+\vec{j}\frac{y}{r}\right)\frac{K}{r^2}$$

<4-140>

$$\vec{e}_r=\frac{\vec{r}}{r}$$

<4-141> (4.7.12)

$$\nabla V(r)=-\vec{e}_r \frac{K}{r^2}$$

<4-142> (4.7.13)

$$\vec{F}=-\nabla V(r)$$

<4-143> (4.7.14)

$$v=\pm\frac{2}{m}\sqrt{E-\frac{\hbar^2}{2mr^2}-\frac{K}{r}}$$

<4-144> (4.7.15)

$$E\geq\frac{\hbar^2}{2mr^2}+\frac{K}{r}$$

<4-145> (4.7.16)

$$Er^2-Kr-\frac{\hbar^2}{2m}\geq 0$$

<4-146>

$$r=\frac{K}{2E}$$

<4-147>

$$\frac{mK^2+\hbar^2E}{2mE}$$

<4-148>

$$D=K^2-\frac{2\hbar^2E}{m}$$

<4-149> (4.7.17)

$$\left\{ \begin{array}{l} r_1 = \frac{1}{2E} \left[K - \sqrt{K^2 - \frac{2h^2 E}{m}} \right] \\ r_2 = \frac{1}{2E} \left[K + \sqrt{K^2 - \frac{2h^2 E}{m}} \right] \end{array} \right.$$

<4-150> (4.7.18)

$$\frac{dr}{dt} = \pm \frac{2}{m} \sqrt{E - \frac{h^2}{2mr^2} - \frac{K}{r}}$$

<4-151> (4.7.19)

$$mr^2 \omega = h$$

<4-152> (4.7.20)

$$r = \frac{p}{1 - e \sin(\theta - \theta_0)}, \quad \left\{ \begin{array}{l} p = \frac{h^2}{mK} \\ e = \sqrt{1 + \frac{2Eh^2}{mK^2}} \end{array} \right.$$

<4-153> (4.8.1)

$$\left\{ \begin{array}{l} \vec{F}_{21} = - \frac{GmM}{|\vec{r}_2 - \vec{r}_1|^2} \left(\vec{r}_2 - \vec{r}_1 \right) \\ \vec{F}_{12} = - \frac{GmM}{|\vec{r}_1 - \vec{r}_2|^2} \left(\vec{r}_1 - \vec{r}_2 \right) \end{array} \right.$$

<4-154> (4.8.2)

$$\left\{ \begin{array}{l} m \frac{d^2 \vec{r}_2}{dt^2} = \vec{F}_{21} \\ M \frac{d^2 \vec{r}_1}{dt^2} = \vec{F}_{12} \end{array} \right.$$

<4-155>

$$m \frac{d^2 \vec{r}}{dt^2} = - \vec{e}_r \frac{GmM}{r^2}$$

<4-156> (4.8.3)

$$E = \frac{m}{2} \left(\frac{dr}{dt} \right)^2 + \frac{h^2}{2mr^2} - \frac{GmM}{r}$$

<4-157> (4.8.4)

$$r = \frac{p}{1 + e \cos \theta}, \quad \text{where } \left\{ \begin{array}{l} \end{array} \right.$$

$$p = \frac{h^2}{Gm^2M}$$

$$e = \sqrt{1 - \frac{2|E|h^2}{G^2m^3M^2}}$$

<4-158> (4.8.5)

$$\left\{ \begin{array}{l} \frac{p}{1-e^2} = \frac{GM}{2|E|} = a \\ \frac{p}{\sqrt{1-e^2}} = \frac{h}{\sqrt{|E|}} = b \\ \frac{ep}{e^2-1} = x_0 \end{array} \right.$$

<4-159> (4.8.6)

$$\frac{(x-x_0)^2}{a^2} + \frac{y^2}{b^2} = 1$$

<4-160> (4.8.7)

$$A = \pi ab$$

<4-161>

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

<4-162>

$$mr^2 \frac{d\theta}{dt} = h$$

<4-163>

$$\frac{dA}{dt} = \frac{h}{2m}$$

<4-164> (4.8.8)

$$T = \frac{A}{(h/2m)} = \frac{2\pi mab}{h}$$

<4-165> (4.8.9)

$$b^2 = \frac{2h^2}{Gm^2M} a$$

<4-166> (4.8.10)

$$T^2 = \frac{8\pi^2}{GM} a^3$$

<4-167> (4.9.1)

$$F = G \frac{Mm}{R^2}$$

<4-168> (4.9.2)

$$\left\{ \begin{array}{l} G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \\ M = 5.97 \times 10^{24} \text{ kg} \\ R = 6.38 \times 10^6 \text{ m} \end{array} \right.$$

$\end{array}\right.$

<4-169> (4.9.3)

$$g = G \frac{M}{R^2} = 9.8 \text{ m/s}^2$$

<4-170> (4.9.4)

$$F = mg$$

<4-171>

$$\pi \times \frac{5}{180} \text{ rad} \approx 0.0873$$

<4-172>

$$\pi \times \frac{10}{180} \text{ rad} \approx 0.1745$$

<4-173>

$$\sin 5^\circ \approx 0.0872$$

<4-174>

$$\sin 10^\circ \approx 0.1736$$

<5-1> (5.1.1)

$\left\{$

$\begin{array}{l}$

$$m_1 \frac{d^2 \vec{r}_1(t)}{dt^2}$$

$$= \vec{F}_{21} (|\vec{r}_2 - \vec{r}_1|)$$

$$m_2 \frac{d^2 \vec{r}_2(t)}{dt^2}$$

$$= \vec{F}_{12} (|\vec{r}_1 - \vec{r}_2|)$$

$\right\}$

<5-2>

$$|\vec{r}_1 - \vec{r}_2| = |\vec{r}_2 - \vec{r}_1| \equiv r$$

<5-3> (5.1.2)

$$\vec{F}_{21}(r) = -\vec{F}_{12}(r)$$

<5-4> (5.1.3)

$\left\{$

$\begin{array}{l}$

$$\vec{p}_1(t) = m_1 \frac{d\vec{r}_1(t)}{dt}$$

$$\equiv m_1 \vec{v}_1(t)$$

$$\vec{p}_2(t) = m_2 \frac{d\vec{r}_2(t)}{dt}$$

$$\equiv m_2 \vec{v}_2(t)$$

$\right\}$

<5-5> (5.1.4)

$$\begin{array}{l} \left\{ \begin{array}{l} \frac{d\vec{p}_1(t)}{dt} = \vec{F}_{21}(r) \\ \frac{d\vec{p}_2(t)}{dt} = \vec{F}_{12}(r) \end{array} \right. \end{array}$$

<5-6> (5.1.5)

$$\frac{d\vec{p}_1(t)}{dt} + \frac{d\vec{p}_2(t)}{dt} = 0$$

<5-7> (5.1.6)

$$\vec{P}(t) = \vec{p}_1(t) + \vec{p}_2(t)$$

<5-8> (5.1.7)

$$\frac{d\vec{P}(t)}{dt} = 0$$

<5-9> (5.1.8)

$$\vec{P}(t) = \vec{P}_0$$

<5-10> (5.1.9)

$$\vec{R}(t) = \frac{m_1\vec{r}_1(t) + m_2\vec{r}_2(t)}{m_1 + m_2}$$

<5-11>

$$\begin{array}{l} \left[\begin{array}{l} \frac{d\vec{R}}{dt} \\ = \frac{m_1}{m_1 + m_2} \frac{d\vec{r}_1}{dt} \\ + \frac{m_2}{m_1 + m_2} \frac{d\vec{r}_2}{dt} \\ = \frac{1}{m_1 + m_2} \left(\vec{p}_1 + \vec{p}_2 \right) \\ = \frac{1}{m_1 + m_2} \vec{P} \end{array} \right. \end{array}$$

<5-12> (5.1.10)

$$\vec{P}(t) = (m_1 + m_2) \frac{d\vec{R}(t)}{dt}$$

<5-13> (5.1.11)

$$\begin{array}{l} \left[\begin{array}{l} m_1 \frac{d^2\vec{r}_1(t)}{dt^2} \\ = \vec{F}_{21}(r_{21}) + \vec{F}_{31}(r_{31}) + \dots + \vec{F}_{N1}(r_{N1}) \\ m_2 \frac{d^2\vec{r}_2(t)}{dt^2} \\ = \vec{F}_{12}(r_{12}) + \vec{F}_{32}(r_{32}) + \dots + \vec{F}_{N2}(r_{N2}) \\ \dots \\ m_N \frac{d^2\vec{r}_N(t)}{dt^2} \\ = \vec{F}_{1N}(r_{1N}) + \vec{F}_{2N}(r_{2N}) + \dots + \vec{F}_{N-1,N}(r_{N-1,N}) \end{array} \right. \end{array}$$

<5-14> (5.1.12)

$$\left\{ \begin{array}{l} \frac{d\vec{r}_1(t)}{dt} \equiv m_1 \vec{v}_1(t) \\ \frac{d\vec{r}_2(t)}{dt} \equiv m_2 \vec{v}_2(t) \\ \dots \\ \frac{d\vec{r}_N(t)}{dt} \equiv m_N \vec{v}_N(t) \end{array} \right.$$

<5-15> (5.1.13)

$$\left\{ \begin{array}{l} \frac{d\vec{p}_1(t)}{dt} = \vec{F}_{21}(r_{21}) + \vec{F}_{31}(r_{31}) + \dots + \vec{F}_{N1}(r_{N1}) \\ \frac{d\vec{p}_2(t)}{dt} = \vec{F}_{12}(r_{12}) + \vec{F}_{32}(r_{32}) + \dots + \vec{F}_{N2}(r_{N2}) \\ \dots \\ \frac{d\vec{p}_N(t)}{dt} = \vec{F}_{1N}(r_{1N}) + \vec{F}_{2N}(r_{2N}) + \dots + \vec{F}_{N-1,N}(r_{N-1,N}) \end{array} \right.$$

<5-16> (5.1.14)

$$\vec{F}_{ij}(r_{ij}) = -\vec{F}_{ji}(r_{ji}) = -\vec{F}_{ji}(r_{ij})$$

<5-17> (5.1.15)

$$\frac{d\vec{P}(t)}{dt} = 0$$

<5-18> (5.1.16)

$$\vec{P}(t) = \vec{p}_1(t) + \vec{p}_2(t) + \dots + \vec{p}_N(t)$$

<5-19> (5.1.17)

$$\vec{R}(t) = \frac{m_1 \vec{r}_1(t) + m_2 \vec{r}_2(t) + \dots + m_N \vec{r}_N(t)}{m_1 + m_2 + \dots + m_N}$$

<5-20> (5.1.18)

$$\vec{P}(t) = (m_1 + m_2 + \dots + m_N) \frac{d\vec{R}(t)}{dt}$$

<5-21> (5.2.1)

$$\vec{r}_1(t) - \vec{r}_2(t) \equiv \vec{r}(t)$$

<5-22> (5.2.2)

$$\left\{ \begin{array}{l} m_1 \frac{d^2 \vec{r}_1(t)}{dt^2} = \vec{F}_{21}(r) \\ m_2 \frac{d^2 \vec{r}_2(t)}{dt^2} = \vec{F}_{12}(r) \end{array} \right.$$

<5-23>

$$\vec{F}_{21}(\mathbf{r}) \equiv \vec{F}(\mathbf{r})$$

<5-24>

$$m_1 m_2 \frac{d^2 \vec{r}(t)}{dt^2} = (m_1 + m_2) \vec{F}(\mathbf{r})$$

<5-25> (5.2.3)

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

<5-26> (5.2.4)

$$\mu \frac{d^2 \vec{r}(t)}{dt^2} = \vec{F}(\mathbf{r})$$

<5-27> (5.2.5)

$$M \frac{d^2 \vec{R}}{dt^2} = 0$$

<5-28>

$$\left[\begin{array}{l} m_1 \frac{d^2 \vec{r}_1(t)}{dt^2} \\ = \vec{F}_{21}(\vec{r}_1 - \vec{r}_2) \\ m_2 \frac{d^2 \vec{r}_2(t)}{dt^2} \\ = \vec{F}_{12}(\vec{r}_2 - \vec{r}_1) \end{array} \right]$$

<5-29>

$$\left[\begin{array}{l} M \frac{d^2 \vec{R}(t)}{dt^2} = 0 \\ \mu \frac{d^2 \vec{r}(t)}{dt^2} = \vec{F}(\mathbf{r}) \end{array} \right]$$

<5-30> (5.2.6)

$$\left[\begin{array}{l} \vec{R}(t) = \frac{m_1 \vec{r}_1(t) + m_2 \vec{r}_2(t)}{m_1 + m_2} \\ \vec{r}(t) = \vec{r}_1(t) - \vec{r}_2(t) \end{array} \right]$$

<5-31>

$$\vec{F}(\mathbf{r}) \equiv \vec{F}_{21}(|\vec{r}_1 - \vec{r}_2|) = -\vec{F}_{12}(|\vec{r}_2 - \vec{r}_1|)$$

<5-32> (5.2.7)

$$\vec{r}_1 = \vec{R} + \frac{m_2}{M} \vec{r}$$

$$\vec{r}_2 = \vec{R} - \frac{m_1}{M} \vec{r}$$

$$\end{array} \right.$$

<5-33> (5.2.8)

$$\begin{array}{l} \frac{m_1 \vec{v}_1^2}{2} + \frac{m_2 \vec{v}_2^2}{2} \\ = \frac{m_1}{2} \left(\frac{d\vec{R}}{dt} \right)^2 \\ + \frac{\vec{m}_2}{M} \frac{d\vec{r}}{dt} \right)^2 \\ + \frac{m_2}{2} \left(\frac{d\vec{R}}{dt} \right)^2 \\ - \frac{\vec{m}_1}{M} \frac{d\vec{r}}{dt} \right)^2 \\ = \frac{M}{2} \left(\frac{d\vec{R}}{dt} \right)^2 \\ + \frac{\mu}{2} \left(\frac{d\vec{r}}{dt} \right)^2 \end{array}$$

<5-34> (5.3.1)

$$\begin{array}{l} m \frac{d^2 x_A}{dt^2} \\ = F_A + f_A \\ = -kx_A + k(x_B - x_A) \\ = -2kx_A + kx_B \end{array}$$

<5-35> (5.3.2)

$$\begin{array}{l} m \frac{d^2 x_B}{dt^2} \\ = F_B + f_B \\ = -kx_B - k(x_B - x_A) \\ = kx_A - 2kx_B \end{array}$$

<5-36>

$$\frac{(\ell + x_A) + (2\ell + x_B)}{2} = \frac{x_A + x_B}{2} + \frac{3}{2} \ell$$

<5-37>

$$(2\ell + x_B) - (\ell + x_A) = x_B - x_A + \ell$$

<5-38> (5.3.3)

$$\left\{ \begin{array}{l} X = \frac{(x_A + \ell) + (x_B + 2\ell)}{2} - \frac{3\ell}{2} \\ = \frac{x_A + x_B}{2} \\ x = (x_B - x_A + \ell) - \ell \\ = x_B - x_A \end{array} \right. \right.$$

<5-39> (5.3.4)

$$M \frac{d^2 X}{dt^2} = -2kX$$

<5-40> (5.3.5)

$$\mu \frac{d^2 x}{dt^2} = -\frac{3k}{2}x$$

<5-41> (5.3.6)

$$X(t) = A \sin(\Omega t + B)$$

<5-42>

$$\Omega = \sqrt{\frac{2k}{M}}$$

<5-43> (5.3.7)

$$x(t) = a \sin(\omega t + b)$$

<5-44>

$$\omega = \sqrt{\frac{3k}{2\mu}}$$

<5-45> (5.3.8)

$$\begin{array}{l} \left\{ \begin{array}{l} x_A = X - \frac{x}{2} \\ x_B = X + \frac{x}{2} \end{array} \right. \\ \text{right.} \end{array}$$

<5-46> (5.3.9)

$$\begin{array}{l} \left\{ \begin{array}{l} x_A = A \sin(\Omega t + B) - \frac{a}{2} \sin(\omega t + b) \\ x_B = A \sin(\Omega t + B) + \frac{a}{2} \sin(\omega t + b) \end{array} \right. \\ \text{right.} \end{array}$$

<5-47> (5.4.1)

$$\begin{array}{l} \left\{ \begin{array}{l} \vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \end{array} \right. \\ \text{right.} \end{array}$$

<5-48> (5.4.2)

$$\begin{array}{l} \left\{ \begin{array}{l} \vec{v} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{M} \end{array} \right. \\ \text{right.} \end{array}$$

<5-49> (5.4.3)

$$\begin{array}{l} \left\{ \begin{array}{l} \vec{v}_1 = \vec{V} + \frac{m_2}{M} \vec{v} \end{array} \right. \\ \text{right.} \end{array}$$

$$\vec{v}_2 = \vec{V} - \frac{m_1}{M} \vec{v}$$

$$\end{array} \right.$$

<5-50> (5.4.4)

$$\begin{array}{l} K = \frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2 \\ = \frac{m_1}{2} \left(\vec{V} + \frac{m_2}{M} \vec{v} \right)^2 + \frac{m_2}{2} \left(\vec{V} - \frac{m_1}{M} \vec{v} \right)^2 \\ = \frac{M}{2} \vec{V}^2 + \frac{m_1 m_2}{2(m_1 + m_2)} v^2 \\ = \frac{M}{2} \vec{V}^2 + \frac{\mu}{2} v^2 \end{array}$$

<5-51> (5.4.5)

$$\left. \begin{array}{l} \vec{v}^{\prime} = \vec{v}_1^{\prime} - \vec{v}_2^{\prime} \\ \vec{V}^{\prime} = \frac{m_1 \vec{v}_1^{\prime} + m_2 \vec{v}_2^{\prime}}{M} \end{array} \right\}$$

<5-52> (5.4.6)

$$\left. \begin{array}{l} v_1^{\prime} = V^{\prime} + \frac{m_2}{M} v^{\prime} \\ v_2^{\prime} = V^{\prime} - \frac{m_1}{M} v^{\prime} \end{array} \right\}$$

<5-53> (5.4.7)

$$\begin{array}{l} K^{\prime} = \frac{m_1}{2} v_1^{\prime 2} + \frac{m_2}{2} v_2^{\prime 2} \\ = \frac{M}{2} V^{\prime 2} + \frac{\mu}{2} v^{\prime 2} \end{array}$$

<5-54>

$$P = m_1 v_1 + m_2 v_2 = M \vec{V}$$

<5-55>

$$P^{\prime} = m_1 v_1^{\prime} + m_2 v_2^{\prime} = M \vec{V}^{\prime}$$

<5-56> (5.4.8)

$$K^{\prime} = \frac{M}{2} V^{\prime 2} + \frac{\mu}{2} v^{\prime 2}$$

<5-57> (5.4.9)

$$K - K^{\prime} = \frac{\mu}{2} v^2$$

$$\left[1 - \frac{1}{2} \left(\frac{v_1^{\prime} - v_2^{\prime}}{v} \right)^2 \right] \left[\frac{1}{2} (v_1 - v_2)^2 \right]$$

<5-58> (5.4.10)

$$e = \frac{|\vec{v}_1^{\prime} - \vec{v}_2^{\prime}|}{|\vec{v}_1 - \vec{v}_2|}$$

<5-59> (5.4.11)

$$K \cdot K^{\prime} = \frac{\mu}{2}(1 - e^2)$$

<5-60> (5.4.12)

$$0 \leq e \leq 1$$

<5-61> (5.4.13)

$$e = \frac{|\vec{v}_1^{\prime} - \vec{v}_2^{\prime}|}{|\vec{v}_1 - \vec{v}_2|}$$

<5-62> (5.4.14)

$\begin{array}{l} \end{array}$

$$m\vec{v}_1 + m\vec{v}_2 = m\vec{v}_1^{\prime} + m\vec{v}_2^{\prime}$$

$$\Leftrightarrow \vec{v}_1 + \vec{v}_2 = \vec{v}_1^{\prime} + \vec{v}_2^{\prime}$$

\end{array}

<5-63> (5.4.15)

$\begin{array}{l} \end{array}$

$$\frac{1}{2}m\vec{v}_1^2 + \frac{1}{2}m\vec{v}_2^2 = \frac{1}{2}m\vec{v}_1^{\prime 2} + \frac{1}{2}m\vec{v}_2^{\prime 2}$$

$$\Leftrightarrow v_1^2 + v_2^2 = v_1^{\prime 2} + v_2^{\prime 2}$$

\end{array}

<5-64> (5.4.16)

$$v_1^{\prime} = v_2^{\prime}$$

<5-65>

$$(v_1 + v_2)^2 - 2v_1v_2 = (v_1^{\prime} + v_2^{\prime})^2 - 2v_1^{\prime}v_2^{\prime}$$

<5-66> (5.4.17)

$$v_1v_2 = v_1^{\prime}v_2^{\prime}$$

<5-67>

$$(v_1 - v_2)^2 = (v_1^{\prime} - v_2^{\prime})^2$$

<5-68> (5.4.18)

$\left\{ \right.$

$\begin{array}{l} \end{array}$

(1) $\& v_1 - v_2 = v_1^{\prime} - v_2^{\prime}$

(2) $\& v_1 - v_2 = -(v_1^{\prime} - v_2^{\prime})$

$\end{array} \right.$

<5-69> (5.4.19)

$$\begin{array}{l} \begin{array}{l} mv_1 + mv_2 = mv_1' + mv_2' \\ & = 2mv_1' \\ & = 2mv_2' \end{array} \end{array}$$

<5-70> (5.4.20)

$$v_1' = v_2' = \frac{v_1 + v_2}{2}$$

<5-71>

$$|\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}}$$

<5-72>

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

<6-1> (6.1.1)

$$\begin{array}{l} |\vec{r}_i - \vec{r}_j| = a \\ \text{quad}(i=1,2,\dots,n; j=1,2,\dots,n; i \neq j) \end{array}$$

<6-2> (6.1.2)

$$\vec{r} = \frac{m(\vec{r}_1 + \vec{r}_2 + \dots + \vec{r}_n)}{M}$$

<6-3> (6.2.1)

$$\begin{array}{l} \left\{ \begin{array}{l} x_k = a_k \cos \theta_k \\ y_k = a_k \sin \theta_k \end{array} \right. \end{array}$$

<6-4> (6.2.2)

$$\vec{v}_k = v_{kx} \vec{i} + v_{ky} \vec{j} + v_{kz} \vec{k}$$

<6-5> (6.2.3)

$$\begin{array}{l} \left\{ \begin{array}{l} \displaystyle v_{kx} = \frac{dx_k}{dt} = -a_k \omega \sin \theta_k \\ \displaystyle v_{ky} = \frac{dy_k}{dt} = a_k \omega \cos \theta_k \\ \displaystyle v_{kz} = \frac{dz_k}{dt} = 0 \end{array} \right. \end{array}$$

<6-6> (6.2.4)

$$v_k = \sqrt{v_{kx}^2 + v_{ky}^2 + v_{kz}^2} = a_k \omega$$

<6-7>

$$\vec{l}_k = \vec{r}_k \times \vec{p}_k$$

<6-8> (6.2.5)

$$\begin{aligned} \vec{l}_k &= m \left[\vec{i} (y_{kv} \cdot z_{kv} - z_{kv} \cdot y_{kv}) + \vec{j} (z_{kv} \cdot x_{kv} - x_{kv} \cdot z_{kv}) \right. \\ &\quad \left. + \vec{k} (x_{kv} \cdot y_{kv} - y_{kv} \cdot x_{kv}) \right] \\ &= \vec{k} (m a_k^2 \omega) \end{aligned}$$

<6-9> (6.2.6)

$$\begin{aligned} \vec{l} &= \sum_{k=1}^n \vec{l}_k \\ &= \vec{k} \left(\sum_{k=1}^n m a_k^2 \omega \right) \end{aligned}$$

<6-10> (6.2.7)

$$I = \sum_{k=1}^n m a_k^2$$

<6-11> (6.2.8)

$$\vec{l} = \vec{k} I \omega$$

<6-12> (6.2.9)

$$\vec{l} = \sum_{k=1}^n \vec{l}_k = I \vec{\omega}$$

<6-13>

$$\vec{l}_k = \vec{r}_k \times \vec{p}_k$$

<6-14>

$$\begin{aligned} \frac{d\vec{l}_k}{dt} &= \frac{d\vec{r}_k}{dt} \times \vec{p}_k + \vec{r}_k \times \frac{d\vec{p}_k}{dt} \end{aligned}$$

<6-15>

$$\frac{d\vec{r}_k}{dt} \times \vec{p}_k = \vec{v}_k \times \vec{p}_k$$

<6-16> (6.2.10)

$$\frac{d\vec{l}_k}{dt} = \vec{r}_k \times \vec{F} \equiv \vec{N}$$

<6-17>

$$I \frac{d\vec{\omega}}{dt} = \frac{d\vec{l}_k}{dt}$$

<6-18> (6.2.11)

$$I \frac{d\vec{\omega}}{dt} = \vec{N}$$

<6-19> (6.2.12)

$$\vec{\omega} = \text{a constant vector}$$

<6-20>

$$\left(\vec{N}_1 = \vec{r}_1 \times \vec{F}_1, \vec{N}_2 = \vec{r}_2 \times \vec{F}_2, \dots, \vec{N}_n = \vec{r}_n \times \vec{F}_n \right)$$

<6-21> (6.2.13)

$$I \frac{d\vec{\omega}}{dt} = \sum_{k=1}^n \left(\vec{r}_k \times \vec{F}_k \right) = \sum_{k=1}^n \vec{N}_k$$

<6-22> (6.3.1)

$$I = I_G + Md^2$$

<6-23> (6.3.2)

$$I_z = I_x + I_y$$

<6-24> (6.4.1)

$$I_1 = M\ell^2$$

<6-25> (6.4.2)

$$\begin{array}{l} \begin{array}{l} I_2 = M \left(\frac{\ell}{2} \right)^2 + M \left(\frac{\ell}{2} \right)^2 \\ = \frac{M\ell^2}{2} \end{array} \\ \end{array}$$

<6-26> (6.4.3)

$$\begin{array}{l} \begin{array}{l} I_2 = I_1 - 2M \left(\frac{\ell}{2} \right)^2 \\ = M\ell^2 - 2M \left(\frac{\ell}{2} \right)^2 \\ = \frac{M\ell^2}{2} \end{array} \\ \end{array}$$

<6-27> (6.4.4)

$$\begin{array}{l} \begin{array}{l} I_1 = m \left[\left(\frac{\ell}{2N} \right)^2 + \left(2 \frac{\ell}{2N} \right)^2 + \dots + \left(2N \frac{\ell}{2N} \right)^2 \right] \\ = m \left(\frac{\ell}{2N} \right)^2 \left[1^2 + 2^2 + \dots + (2N)^2 \right] \\ = \frac{m\ell^2}{4N^2} \times \frac{(2N)(2N+1)(4N+1)}{6} \\ = \frac{M\ell^2}{8N^3} \times \frac{(2N)(2N+1)(4N+1)}{6} \\ \quad (\text{because } 2m = M) \end{array} \\ \end{array}$$

<6-28> (6.4.5)

$$\sum_{k=1}^N k^2 = \frac{N(N+1)(2N+1)}{6}$$

<6-29>

$$I_1 = \frac{M \ell^2}{8} \frac{2 \left\{ 2 + \frac{1}{N} \right\} \left\{ 4 + \frac{1}{N} \right\}}{6}$$

<6-30>

$$I_1 = \frac{1}{3} M \ell^2$$

<6-31> (6.4.7)

$\begin{array}{l} \\ \\ \end{array}$

$$I_2 = \frac{2m \ell^2}{8N^3} \left[\left(\frac{\ell}{2N} \right)^2 + \left(\frac{\ell}{2N} \right)^2 + \dots + \left(\frac{\ell}{2N} \right)^2 \right]$$

$$= \frac{2m \ell^2}{8N^3} \times \frac{(2N)(2N+1)(4N+1)}{6}$$

$$= \frac{2m \ell^2}{8N^3} \times \frac{(2N)(2N+1)(4N+1)}{6}$$

quad (because $2m = M$)

\end{array}

<6-32> (6.4.8)

$$I_2 = \frac{1}{12} M \ell^2$$

<6-33> (6.4.9)

$$I_1 = I_2 + M \left(\frac{\ell}{2} \right)^2$$

<6-34> (6.4.10)

$$I_2 = I_1 - M \left(\frac{\ell}{2} \right)^2 = \frac{1}{12} M \ell^2$$

<6-35>

$$\frac{1}{3} M \ell^2$$

<6-36>

$$\frac{1}{12} M \ell^2$$

<6-37>

$$M \ell^2$$

<6-38>

$$\frac{1}{2} M \ell^2$$

<6-39>

$$\frac{1}{8} M b^2$$

<6-40>

$$\frac{M}{8} (a^2 + b^2)$$

<6-41>

$$\forall \text{frac}\{M\}\{4\}a^2$$

<6-42>

$$\forall \text{frac}\{M\}\{2\}a^2$$

<6-43>

$$\forall \text{frac}\{2M\}\{5\}a^2$$

<6-44>

$$\forall \text{frac}\{M\}\{4\}a^2 + \forall \text{frac}\{M\}\{3\}h^2$$

<6-45>

$$\forall \text{frac}\{M\}\{2\}a^2$$