

<2-1> (2.2.1)

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

<2-2> (2.2.2)

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

<2-3>

$$x = \frac{x^{\prime}}{\sqrt{a}}$$

<2-4>

$$dx = \frac{1}{\sqrt{a}} dx^{\prime}$$

<2-5>

$$x^{\prime} \sqrt{a} = x$$

<2-6>

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-x^{\prime 2}} dx^{\prime}$$

<2-7>

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \frac{1}{\sqrt{a}} \sqrt{\pi} = \sqrt{\frac{\pi}{a}}$$

<2-8>

$$\begin{array}{l} \displaystyle \frac{d}{da} \left( \int_{-\infty}^{\infty} e^{-ax^2} dx \right) = \displaystyle \int_{-\infty}^{\infty} \frac{d}{da} \left( e^{-ax^2} \right) dx \\ \quad = \displaystyle \int_{-\infty}^{\infty} x^2 e^{-ax^2} dx \end{array}$$

<2-8>

$$\begin{array}{l} \displaystyle \frac{d}{da} \left( \int_{-\infty}^{\infty} e^{-ax^2} dx \right) = \displaystyle \int_{-\infty}^{\infty} \frac{d}{da} \left( e^{-ax^2} \right) dx \\ \quad = \displaystyle \int_{-\infty}^{\infty} x^2 e^{-ax^2} dx \end{array}$$

<2-9>

$$\begin{array}{l} \displaystyle \frac{d}{da} \left( \sqrt{\frac{\pi}{a}} \right) = \displaystyle \sqrt{\pi} \frac{d}{da} \left( a^{-1/2} \right) \\ \quad = \displaystyle -\sqrt{\pi} \frac{a^{-3/2}}{2} \end{array}$$

<2-10>

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2} a^{-3/2}$$

<2-11>

$$\begin{array}{l} \displaystyle \int_{-\infty}^{\infty} x^4 e^{-ax^2} dx \\ = \displaystyle \left( \frac{\sqrt{\pi}}{2} \cdot \frac{3}{2} \right) a^{-5/2} \\ = \displaystyle \frac{1 \cdot 3 \sqrt{\pi}}{2^2} a^{-5/2} \end{array}$$

<2-12>

$$\begin{array}{l} \displaystyle \int_{-\infty}^{\infty} x^6 e^{-ax^2} dx \\ = \displaystyle \left( \frac{\sqrt{\pi}}{2} \cdot \frac{3}{2} \cdot \frac{5}{2} \right) a^{-7/2} \\ = \displaystyle \frac{1 \cdot 3 \cdot 5 \sqrt{\pi}}{2^3} a^{-7/2} \end{array}$$

<2-13>

$$\begin{aligned} \int_{-\infty}^{\infty} x^{2n} e^{-ax^2} dx \\ = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) \sqrt{\pi}}{2^n} a^{-(2n+1)/2} \quad \text{quad} \\ (n=0,1,2,\cdots) \end{aligned}$$

<2-14> (2.2.3)

$$\begin{aligned} \int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx \\ = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) \sqrt{\pi}}{2^n} \quad \text{quad} \\ (n=0,1,2,\cdots) \end{aligned}$$

<2-15>

$$x^{2n} e^{-x^2} \equiv f(x)$$

<2-16>

$$f(-x) = f(x)$$

<2-17>

$$\begin{array}{l} \displaystyle \int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx \\ = \displaystyle \int_{-\infty}^0 x^{2n} e^{-x^2} dx + \int_0^{\infty} x^{2n} e^{-x^2} dx \\ = \displaystyle 2 \int_0^{\infty} x^{2n} e^{-x^2} dx \end{array}$$

<2-18> (2.2.4)

$$\begin{aligned} \int_0^{\infty} x^{2n} e^{-x^2} dx \\ = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1) \sqrt{\pi}}{2^{n+1}} \quad \text{quad } (n=0,1,2,\cdots) \end{aligned}$$

<2-19>

$$xe^{-ax^2} \equiv g(x)$$

<2-20>

$$g(-x) = -g(x)$$

<2-21>

$$\int_{-\infty}^{\infty} xe^{-ax^2} dx = 0$$

<2-22> (2.2.5)

$$\int_{-\infty}^{\infty} x^{2n+1} e^{-x^2} dx = 0 \quad (n=0, 1, 2, \dots)$$

<2-23>

$$\int_0^{\infty} xe^{-ax^2} dx$$

<2-24>

$$x = \sqrt{\frac{y}{a}}$$

<2-25>

$$dx = \frac{1}{2\sqrt{ay}} dy$$

<2-26>

$\begin{array}{l}$

$$\int_0^{\infty} xe^{-ax^2} dx$$

$$= \int_0^{\infty} \sqrt{\frac{y}{a}} \frac{1}{2\sqrt{ay}} dy = \frac{1}{2a} \int_0^{\infty} e^{-y} dy$$

$$= \frac{1}{2a} \int_0^{\infty} e^{-y} dy$$

$\end{array}$

<2-27>

$$\int_0^{\infty} e^{-y} dy = 1$$

<2-28>

$$\int_0^{\infty} xe^{-ax^2} dx = \frac{1}{2a}$$

<2-29>

$$\int_0^{\infty} x^3 e^{-ax^2} dx = \frac{1}{2a^2}$$

<2-30>

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

<2-31> (2.2.6)

$$\int_0^{\infty} x^{2n+1} e^{-x^2} dx = \frac{n!}{2} \quad (n=0, 1, 2, \dots)$$

<2-32>

$$\begin{aligned} & \int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots \sqrt{\pi}}{2^n} \\ & (2.2.3) \\ & \int_0^{\infty} x^{2n} e^{-x^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots \sqrt{\pi}}{2^{n+1}} \\ & (2.2.4) \\ & \int_{-\infty}^{\infty} x^{2n+1} e^{-x^2} dx = 0 \\ & (2.2.5) \\ & \int_0^{\infty} x^{2n+1} e^{-x^2} dx = \frac{n!}{2} \\ & (2.2.6) \end{aligned}$$

<2-33>

$$dv_x dv_y dv_z \equiv d\Gamma$$

<2-34> (2.2.7)

$$\begin{aligned} & \begin{aligned} & P(v_x, v_y, v_z) d\Gamma = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-E/kT} d\Gamma \\ & = \left( \frac{m}{2\pi kT} \right)^{3/2} \exp \left[ -\frac{mv_x^2}{2kT} \right] \exp \left[ -\frac{mv_y^2}{2kT} \right] \exp \left[ -\frac{mv_z^2}{2kT} \right] d\Gamma \end{aligned} \\ & \end{aligned}$$

<2-35> (2.2.8)

$$E = \frac{m}{2} (v_x^2 + v_y^2 + v_z^2)$$

<2-36>

$$\begin{aligned} & \begin{aligned} & -\infty \leq v_x \leq \infty \\ & -\infty \leq v_y \leq \infty \\ & -\infty \leq v_z \leq \infty \end{aligned} \\ & \end{aligned}$$

<2-37>

$$\iiint P(v_x, v_y, v_z) d\Gamma = 1$$

<2-38>

$$\langle K \rangle = \frac{3kT}{2}$$

<2-39>

$$K = \frac{m}{2} (v_x^2 + v_y^2 + v_z^2)$$

<2-40>

$$\sqrt{v_x^2 + v_y^2 + v_z^2}$$

<2-41> (2.2.9)

$$\begin{array}{l} \displaystyle \angle K \\ \displaystyle \int P(v_x, v_y, v_z) \frac{m^3}{2\pi kT} \frac{d\Gamma}{d\Gamma} \\ \displaystyle \left( \frac{m}{2\pi kT} \right)^{3/2} \int_{-\infty}^{\infty} \frac{m^3}{2} e^{-(mv_x^2/2)/kT} dv_x \\ \displaystyle \times \int_{-\infty}^{\infty} e^{-(mv_y^2/2)/kT} dv_y \\ \displaystyle \times \int_{-\infty}^{\infty} e^{-(mv_z^2/2)/kT} dv_z \end{array}$$

<2-42> (2.2.10)

$$\begin{array}{l} \left[ \begin{array}{l} v_x = \left( \frac{2kT}{m} \right)^{1/2} p \\ v_y = \left( \frac{2kT}{m} \right)^{1/2} q \\ v_z = \left( \frac{2kT}{m} \right)^{1/2} r \end{array} \right] \\ \end{array}$$

<2-43> (2.2.11)

$$\begin{array}{l} \left[ \begin{array}{l} p = \left( \frac{m}{2kT} \right)^{1/2} v_x \\ q = \left( \frac{m}{2kT} \right)^{1/2} v_y \\ r = \left( \frac{m}{2kT} \right)^{1/2} v_z \end{array} \right] \\ \end{array}$$

<2-44> (2.2.12)

$$\begin{array}{l} \displaystyle \angle K \\ \displaystyle \int \left( \frac{m}{2\pi kT} \right)^{3/2} \times \left( \frac{m}{2\pi kT} \right)^{1/2} \\ \displaystyle \int_{-\infty}^{\infty} \left( \frac{m}{2\pi kT} \right)^{1/2} p^2 dp \\ \displaystyle \times \int_{-\infty}^{\infty} e^{-q^2} dq \\ \displaystyle \times \int_{-\infty}^{\infty} e^{-r^2} dr \end{array}$$

<2-45>

$$\int_{-\infty}^{\infty} p^2 e^{-p^2} dp = \frac{\sqrt{\pi}}{2}$$

<2-46>

$$\int_{-\infty}^{\infty} e^{-q^2} dq = \int_{-\infty}^{\infty} e^{-r^2} dr = \sqrt{\pi}$$

<2-47> (2.2.13)

$$\begin{array}{l} \angle K \angle \\ = \left( \frac{kT}{\pi} \right)^{3/2} \times \left( \frac{\sqrt{\pi}}{2} \right) \\ \times \left( \sqrt{\pi} \right)^2 \\ = \frac{3kT}{2} \\ \end{array}$$

<2-48>

$$\vec{v} = \vec{i}v_x + \vec{j}v_y + \vec{k}v_z$$

<2-49> (2.2.14)

$$\begin{array}{l} \displaystyle \angle \vec{v} \angle \\ = \left( \frac{m}{2\pi kT} \right)^{3/2} \\ \left[ \int_{-\infty}^{\infty} v_x e^{-(mv_x^2/2)/kT} dv_x \right] \\ \times \left[ \int_{-\infty}^{\infty} v_y e^{-(mv_y^2/2)/kT} dv_y \right] \\ \times \left[ \int_{-\infty}^{\infty} v_z e^{-(mv_z^2/2)/kT} dv_z \right] \\ + \left( \frac{m}{2\pi kT} \right)^{3/2} \\ \left[ \int_{-\infty}^{\infty} e^{-(mv_x^2/2)/kT} dv_x \right] \\ \times \left[ \int_{-\infty}^{\infty} v_y e^{-(mv_y^2/2)/kT} dv_y \right] \\ \times \left[ \int_{-\infty}^{\infty} v_z e^{-(mv_z^2/2)/kT} dv_z \right] \\ + \left( \frac{m}{2\pi kT} \right)^{3/2} \\ \left[ \int_{-\infty}^{\infty} e^{-(mv_x^2/2)/kT} dv_x \right] \\ \times \left[ \int_{-\infty}^{\infty} v_y e^{-(mv_y^2/2)/kT} dv_y \right] \\ \times \left[ \int_{-\infty}^{\infty} v_z e^{-(mv_z^2/2)/kT} dv_z \right] \\ \end{array}$$

<2-50> (2.2.15)

$$\angle \vec{v} \angle = 0$$

<2-51>

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

<2-52> (2.2.16)

$$\begin{array}{l} \displaystyle \angle v \angle \\ = \left( \frac{m}{2\pi kT} \right)^{3/2} \int_{-\infty}^{\infty} \\ e^{-(mv_x^2/2)/kT} dv_x \int_{-\infty}^{\infty} e^{-(mv_y^2/2)/kT} dv_y \\ \times \int_{-\infty}^{\infty} \sqrt{v_x^2 + v_y^2 + v_z^2} \\ e^{-(mv_z^2/2)/kT} dv_z \\ \end{array}$$

<2-53>

$$\begin{array}{l} x=r\sin\theta\cos\phi \\ y=r\sin\theta\sin\phi \\ z=r\cos\theta \end{array}$$

<2-54> (2.2.17)

$$\begin{array}{l} v_x=v\sin\theta\cos\phi \\ v_y=v\sin\theta\sin\phi \\ v_z=v\cos\theta \end{array}$$

<2-55> (2.2.18)

$$\begin{array}{l} \displaystyle \langle v \rangle \\ = \displaystyle \left( \frac{m}{2\pi kT} \right)^{3/2} \int_0^\infty v^2 dv \\ \int_{-\pi}^\pi \sin\theta d\theta \int_0^{2\pi} v e^{-(mv^2/2)/kT} d\phi \\ = \displaystyle \left( \frac{m}{2\pi kT} \right)^{3/2} \int_0^\infty v^3 \\ e^{-(mv^2/2)/kT} dv \int_{-\pi}^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \\ = \displaystyle \left( \frac{m}{2\pi kT} \right)^{3/2} 4\pi \int_0^\infty v^3 \\ e^{-(mv^2/2)/kT} dv \\ = \displaystyle \sqrt{\frac{8kT}{\pi m}} \end{array}$$

<2-56>

$$K=\frac{m}{2} \left( v_x^2 + v_y^2 + v_z^2 \right) = \frac{m}{2} v^2$$

<2-57>

$$\langle v^2 \rangle = \frac{3kT}{m}$$

<2-58>

$$\langle v \rangle^2 = \frac{8kT}{m} \simeq \frac{2,55kT}{m}$$

<2-59>

$$\sqrt{\langle v^2 \rangle - \langle v \rangle^2}$$

<2-60> (2.3.1)

$$P(\vec{v}; T) d\vec{v} = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\beta \epsilon(\vec{v})} d\vec{v}$$

<2-61> (2.3.2)

$$\beta = \frac{1}{kT}$$

<2-62> (2.3.3)

$$\epsilon(\vec{v}) = \frac{m}{2} \vec{v}^2 \\ = \frac{m}{2} v_x^2 + \frac{m}{2} v_y^2 + \frac{m}{2} v_z^2$$

<2-63> (2.3.4)

$$\iiint P(\vec{v}; T) d\vec{v} = 1$$

<2-64> (2.3.5)

$$P(\{\vec{v}\}; T) d\vec{v}_1 d\vec{v}_2 \dots d\vec{v}_N \propto e^{-\beta E(\{\vec{v}\})} \\ d\vec{v}_1 d\vec{v}_2 \dots d\vec{v}_N$$

<2-65> (2.3.6)

$$E(\{\vec{v}\}) = \frac{m}{2} (v_1^2 + v_2^2 + v_N^2)$$

<2-66> (2.3.7)

$$P(\{\vec{v}\}; T) d\vec{v}_1 d\vec{v}_2 \dots d\vec{v}_N = C W [E(\{\vec{v}\})] e^{-\beta E(\{\vec{v}\})} d\vec{v}_1 d\vec{v}_2 \dots d\vec{v}_N$$

<2-67> (2.3.8)

$$\iiint P(\{\vec{v}\}) d\vec{v}_1 d\vec{v}_2 \dots d\vec{v}_N = 1$$

<2-68> (2.3.9)

$$P[\{\vec{r}\}, \{\vec{v}\}; T] d\Gamma \\ = C W [E(\{\vec{r}\}, \{\vec{v}\})] e^{-\beta E(\{\vec{r}\}, \{\vec{v}\})} d\Gamma$$

<2-69> (2.3.10)

$$d\vec{r}_1 d\vec{r}_2 \dots d\vec{r}_N d\vec{v}_1 d\vec{v}_2 \dots d\vec{v}_N \\ \equiv d\Gamma$$

<2-70> (2.3.11)

$$\iiint_{VP} [\{\vec{r}\}, \{\vec{v}\}; T] d\Gamma = 1$$

<2-71> (2.3.12)

$$W [E(\{\vec{r}\}, \{\vec{v}\})] e^{-\beta E(\{\vec{r}\}, \{\vec{v}\})}$$

<2-72> (2.3.13)

$$x = e^{\ln x}$$

<2-73> (2.3.14)

$$\begin{array}{l} \\ \displaystyle{W e^{-\beta E}} = \displaystyle{e^{\ln W} e^{-\beta E}} \\ \displaystyle{=} \displaystyle{e^{-\beta E - kT \ln W}} \\ \end{array}$$



<2-74> (2.3.15)

$$\begin{array}{l} \\ \displaystyle{\frac{1}{\iint_V \exp\left[-\beta \right.} \\ \left. E(\vec{r}), \vec{v}\right) \cdot kT \ln W(\vec{r}, \vec{v}) \right] d\Gamma} \\ \\ \displaystyle{\frac{1}{Z(T)}} \\ \end{array}$$

<2-75> (2.3.16)

$$Z(T) \equiv \iint_V e^{\{-\beta [E(\vec{r}), \vec{v}) - kT \ln W(\vec{r}, \vec{v})]\}} d\Gamma$$

<2-76> (2.3.17)

$$\iint_V E(\vec{r}, \vec{v}) P[\vec{r}, \vec{v}; T] d\Gamma \equiv U$$

<2-77> (2.3.18)

$$k \iint_V \ln W(\vec{r}, \vec{v}) P[\vec{r}, \vec{v}; T] d\Gamma \equiv S$$

<2-78>

$$E - kT \ln W$$

<2-79>

$$U - ST \equiv F(T, V)$$

<2-80> (2.3.20)

$$\frac{d e^{\{ax\}}}{dx} = a e^{\{ax\}}$$

<2-81> (2.3.21)

$$\frac{dZ(T)}{d\beta} = - \iint_V E(\vec{r}, \vec{v}) e^{\{-\beta E(\vec{r}, \vec{v})\}} d\Gamma$$

<2-82> (2.3.22)

$$\begin{array}{l} \\ \displaystyle{\frac{1}{Z(T)} \frac{dZ(T)}{d\beta}} \\ \\ \displaystyle{\iint_V E(\vec{r}, \vec{v})} \\ \displaystyle{\frac{e^{\{-\beta E(\vec{r}, \vec{v})\}}}{Z(T)} d\Gamma} \\ \\ \displaystyle{- \iint_V} \\ E(\vec{r}, \vec{v}) P\left[\vec{r}, \vec{v}; T\right] d\Gamma} \\ \end{array}$$

<2-83> (2.3.23)

$$\begin{array}{l} \\ \displaystyle{\frac{1}{Z(T)} \frac{dZ(T)}{d\beta}} \\ \\ \displaystyle{\frac{d \ln Z(T)}{d\beta}} \end{array}$$

$\forall \text{end}\{\text{array}\}$

<2-84> (2.3.24)

$$\forall \text{frac}\{d\forall \ln y(x)\}\{dx\}=\forall \text{frac}\{1\}\{y\}\forall \text{frac}\{dy\}\{dx\}$$

<2-85>

$$f(-x)=f(x)$$

<2-86>

$$I=\forall \text{int}_{\{-\forall \text{infty}\}^{\{\forall \text{infty}\}}}f(x)dx$$

<2-87>

$$I=\forall \text{int}_{\{-\forall \text{infty}\}^{\{0\}}}f(x)dx+\forall \text{int}_{\{0\}^{\{\forall \text{infty}\}}}f(x)dx$$

<2-88>

$$dx=-dx^{\forall \text{prime}}$$

<2-89>

$$\begin{aligned}\forall \text{int}_{\{-\forall \text{infty}\}^{\{0\}}}f(x)dx &= \forall \text{int}_{\{+\forall \text{infty}\}^{\{0\}}}f(-x^{\forall \text{prime}})(-dx^{\forall \text{prime}}) \\ &= -\forall \text{int}_{\{+\forall \text{infty}\}^{\{0\}}}f(x^{\forall \text{prime}})dx^{\forall \text{prime}} \\ &+ \forall \text{int}_{\{0\}^{\{\forall \text{infty}\}}}f(x)dx\end{aligned}$$

<2-90>

$$\forall \text{int}_{\{a\}^{\{b\}}}F(x)dx = -\forall \text{int}_{\{b\}^{\{a\}}}F(x)dx$$

<2-91>

$$-\forall \text{int}_{\{-\forall \text{infty}\}^{\{0\}}}f(x)dx = \forall \text{int}_{\{0\}^{\{+\forall \text{infty}\}}}f(x^{\forall \text{prime}})dx^{\forall \text{prime}} = \forall \text{int}_{\{0\}^{\{+\forall \text{infty}\}}}f(x)dx$$

<2-92>

$$\forall \text{int}_{\{-\forall \text{infty}\}^{\{0\}}}f(x)dx = 2\forall \text{int}_{\{0\}^{\{\forall \text{infty}\}}}f(x)dx$$

<2-93>

$\forall \text{begin}\{\text{array}\}\{\}$

$$\forall \text{displaystyle}\{\forall \text{left}(\forall \text{frac}\{m\}\{2kT\}\forall \text{right})^{\{3/2\}}\forall \text{left}(\forall \text{int}_{\{-\forall \text{infty}\}^{\{\forall \text{infty}\}}}e^{\{-(mv_x^2/2)/kT\}}dv_x\forall \text{right})\}$$

$$\forall \text{times}\forall \text{left}(\forall \text{int}_{\{-\forall \text{infty}\}^{\{\forall \text{infty}\}}}e^{\{-(mv_y^2/2)/kT\}}dv_y\forall \text{right})\}\ \forall \forall \ \forall \forall$$

$$\&\forall \text{qqquad}\forall \text{times}\forall \text{displaystyle}\{\forall \text{left}(\forall \text{int}_{\{-\forall \text{infty}\}^{\{\forall \text{infty}\}}}e^{\{-(mv_z^2/2)/kT\}}dv_z\forall \text{right})\}=1$$

$\forall \text{end}\{\text{array}\}$

<2-94>

$$\forall \text{left}(\forall \text{frac}\{m\}\{2kT\}\forall \text{right})^{\{3/2\}}$$

$$\forall \text{left}(\forall \text{int}_{\{-\forall \text{infty}\}^{\{\forall \text{infty}\}}}e^{\{-(mv_x^2/2)/kT\}}dv_x\forall \text{right})^3=1$$

<2-95>

$$q=\forall \text{left}(\forall \text{frac}\{m\}\{2kT\}\forall \text{right})^{\{1/2\}}v_x$$

<2-96>

$$dv_x = \sqrt{\frac{2kT}{m}} dq$$

<2-97>

$$\begin{aligned} \text{the left side} &= \sqrt{\frac{m}{2\pi kT}}^3 \\ &\int_{-\infty}^{\infty} \sqrt{\frac{2kT}{m}}^3 \int_{-\infty}^{\infty} e^{-q^2} dq \end{aligned}$$

<2-98>

$$\begin{aligned} \text{the left side} &= \sqrt{\frac{m}{2\pi}}^3 \\ &\int_{-\infty}^{\infty} \sqrt{\frac{2kT}{m}}^3 \sqrt{\pi} dq = 1 \end{aligned}$$

<2-99>

$$\sqrt{\frac{mN^{2/3}}{2\pi kT}}^3$$