

<2-1> (2.2.1)

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

<2-2> (2.2.2)

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

<2-3>

$$x = \frac{x' \sqrt{a}}{\sqrt{a}}$$

<2-4>

$$dx = \frac{1}{\sqrt{a}} dx'$$

<2-5>

$$x' \left( = \sqrt{a} x \right)$$

<2-6>

$$\int_{-\infty}^{\infty} e^{-ax^2} dx$$

$$= \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-x'^2} dx'$$

<2-7>

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \frac{1}{\sqrt{a}} \sqrt{\pi} = \sqrt{\frac{\pi}{a}}$$

<2-8>

$\begin{array}{l} \end{array}$

$$\begin{aligned} & \frac{d}{da} \left( \int_{-\infty}^{\infty} e^{-ax^2} dx \right) \\ &= \frac{d}{da} \left( \int_{-\infty}^{\infty} e^{-x'^2} dx' \right) \\ &= - \int_{-\infty}^{\infty} x' e^{-x'^2} dx' \end{aligned}$$

<2-8>

$\begin{array}{l} \end{array}$

$$\begin{aligned} & \frac{d}{da} \left( \int_{-\infty}^{\infty} e^{-ax^2} dx \right) \\ &= \frac{d}{da} \left( \int_{-\infty}^{\infty} e^{-x'^2} dx' \right) \\ &= - \int_{-\infty}^{\infty} x' e^{-x'^2} dx' \end{aligned}$$

<2-9>

$\begin{array}{l} \end{array}$

$$\begin{aligned} & \frac{d}{da} \left( \int_{-\infty}^{\infty} e^{-ax^2} dx \right) \\ &= \frac{d}{da} \left( \int_{-\infty}^{\infty} e^{-x'^2} dx' \right) \\ &= - \int_{-\infty}^{\infty} x' e^{-x'^2} dx' \end{aligned}$$

<2-10>

$$\int_{-\infty}^{\infty} x^2 e^{-ax^2} dx = \frac{\sqrt{\pi}}{2} a^{-3/2}$$

<2-11>

$$\begin{aligned} &\begin{array}{l} \text{\$begin\{array\}\{r\}} \\ \text{\$displaystyle\{\int_{-\infty}^{\infty} x^4 e^{-ax^2} dx\}} \\ \&=\text{\$displaystyle\{\left(\frac{\sqrt{\pi}}{2}\right)^2 a^{-5/2}\}} \quad \text{\$\\\$\\\$} \\ \&=\text{\$displaystyle\{\frac{1}{3} \cdot 3 \sqrt{\pi} a^{-5/2}\}} \\ \text{\$end\{array\}} \end{array} \end{aligned}$$

<2-12>

$$\begin{aligned} &\begin{array}{l} \text{\$begin\{array\}\{r\}} \\ \text{\$displaystyle\{\int_{-\infty}^{\infty} x^6 e^{-ax^2} dx\}} \\ \&=\text{\$displaystyle\{\left(\frac{\sqrt{\pi}}{2}\right)^3 \cdot 5 \cdot \frac{1}{2} a^{-7/2}\}} \quad \text{\$\\\$\\\$} \\ \&=\text{\$displaystyle\{\frac{1}{4} \cdot 3 \cdot 5 \sqrt{\pi} a^{-7/2}\}} \\ \text{\$end\{array\}} \end{array} \end{aligned}$$

<2-13>

$$\begin{aligned} &\int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx \\ &= \frac{1}{2} \cdot 3 \cdot 5 \cdots (2n-1) \sqrt{\pi} \cdot 2^n a^{-(2n+1)/2} \quad \text{\$quad} \\ &(n=0,1,2,\dots) \end{aligned}$$

<2-14> (2.2.3)

$$\begin{aligned} &\int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx \\ &= \frac{1}{2} \cdot 3 \cdot 5 \cdots (2n-1) \sqrt{\pi} \cdot 2^n \quad \text{\$quad} \\ &(n=0,1,2,\dots) \end{aligned}$$

<2-15>

$$x^{2n} e^{-x^2} \equiv f(x)$$

<2-16>

$$f(-x) = f(x)$$

<2-17>

$$\begin{aligned} &\begin{array}{l} \text{\$begin\{array\}\{r\}} \\ \text{\$displaystyle\{\int_{-\infty}^{\infty} x^{2n} e^{-x^2} dx\}} \\ \&=\text{\$displaystyle\{\int_{-\infty}^0 x^{2n} e^{-x^2} dx + \int_0^{\infty} x^{2n} e^{-x^2} dx\}} \\ \&=\text{\$displaystyle\{2 \int_0^{\infty} x^{2n} e^{-x^2} dx\}} \\ \text{\$end\{array\}} \end{array} \end{aligned}$$

<2-18> (2.2.4)

$$\begin{aligned} &\int_0^{\infty} x^{2n} e^{-x^2} dx \\ &= \frac{1}{2} \cdot 3 \cdot 5 \cdots (2n-1) \sqrt{\pi} \cdot 2^{n+1} \quad \text{\$quad} \quad (n=0,1,2,\dots) \end{aligned}$$

<2-19>

$$xe^{-ax^2} \equiv g(x)$$

<2-20>

$$g(-x) = -g(x)$$

<2-21>

$$\int_{-\infty}^{\infty} xe^{-ax^2} dx = 0$$

<2-22> (2.2.5)

$$\int_{-\infty}^{\infty} x^{2n+1} e^{-x^2} dx = 0 \quad \text{quad}(n=0,1,2,\dots)$$

<2-23>

$$\int_0^{\infty} xe^{-ax^2} dx$$

<2-24>

$$x = \sqrt{\frac{y}{a}}$$

<2-25>

$$dx = \frac{1}{2\sqrt{ay}} dy$$

<2-26>

$$\begin{aligned} & \int_0^{\infty} xe^{-ax^2} dx \\ &= \int_0^{\infty} \sqrt{\frac{y}{a}} e^{-ay} \frac{1}{2\sqrt{ay}} dy \\ &= \frac{1}{2a} \int_0^{\infty} e^{-ay} dy \end{aligned}$$

<2-27>

$$\int_0^{\infty} e^{-y} dy = 1$$

<2-28>

$$\int_0^{\infty} xe^{-ax^2} dx = \frac{1}{2a}$$

<2-29>

$$\int_0^{\infty} x^3 e^{-ax^2} dx = \frac{1}{2a^2}$$

<2-30>

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}$$

<2-31> (2.2.6)

$$\int_0^{\infty} x^{2n+1} e^{-x^2} dx = \frac{n!}{2} \quad \text{quad}(n=0,1,2,\dots)$$

<2-32>

```
¥left¥{¥begin{array}{l}
¥displaystyle{¥int_{-\infty}^{\infty}x^{2n}e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}3\cdot5\cdots(2n+1)\cdot(2n-1)\cdots1}
&(2.2.3) ¥¥ ¥¥
¥displaystyle{¥int_0^{\infty}x^{2n}e^{-x^2}dx=\frac{1}{2}\sqrt{\pi}3\cdot5\cdots(2n+1)\cdot(2n-1)\cdots1}
&(2.2.4) ¥¥ ¥¥
¥displaystyle{¥int_{-\infty}^{\infty}x^{2n+1}e^{-x^2}dx=0}
&(2.2.5) ¥¥ ¥¥
¥displaystyle{¥int_0^{\infty}x^{2n+1}e^{-x^2}dx=\frac{n!}{2}}
&(2.2.6)
¥end{array}}¥right.¥quad(n=0,1,2,\ldots)
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<2-33>

```
dv_xdv_ydv_z¥equiv d¥Gamma
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<2-34> (2.2.7)

```
¥begin{array}{r}
P(v_x,v_y,v_z)d¥Gamma=\frac{m}{2\pi kT}e^{-\frac{mv_x^2}{2kT}}\cdot e^{-\frac{mv_y^2}{2kT}}\cdot e^{-\frac{mv_z^2}{2kT}}
=\frac{m}{2\pi kT}\left(\frac{m}{2\pi kT}\right)^{3/2}e^{-\frac{mv_x^2+mv_y^2+mv_z^2}{2kT}}d¥Gamma
\end{array}
```

<2-35> (2.2.8)

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E=\frac{m}{2}(v_x^2+v_y^2+v_z^2)
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<2-36>

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¥left¥{¥begin{array}{l}
-\infty\leq v_x\leq\infty ¥¥ ¥¥
-\infty\leq v_y\leq\infty ¥¥ ¥¥
-\infty\leq v_z\leq\infty
¥end{array}}¥right.
```

<2-37>

```
¥iint P(v_x,v_y,v_z)d¥Gamma=1
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<2-38>

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¥langle K¥rangle=\frac{3kT}{2}
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<2-39>

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K=\frac{m}{2}(v_x^2+v_y^2+v_z^2)
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<2-40>

$$\left\langle v_x^2, v_y^2, v_z^2 \right\rangle$$

<2-41> (2.2.9)

$$\begin{aligned} &\text{\$begin\{array\}\{rl\}} \\ &\text{\$displaystyle\{\$angle K\$rangle} \\ &\&=\text{\$displaystyle\{3\$left(\$frac\{m\}\{2\$pi kT\}\$right)\^{3/2}\$left(\$int_{-\infty}^{\infty}\$e^{\{-mv_x^2/2\}/kT}\$dv_x\$\right)\}} \quad \text{\$} \\ &\&=\text{\$displaystyle\{3\$left(\$frac\{m\}\{2\$pi kT\}\$right)\^{3/2}\$left(\$int_{-\infty}^{\infty}\$e^{\{-mv_y^2/2\}/kT}\$dv_y\$\right)\}} \quad \text{\$} \\ &\&=\text{\$displaystyle\{3\$left(\$frac\{m\}\{2\$pi kT\}\$right)\^{3/2}\$left(\$int_{-\infty}^{\infty}\$e^{\{-mv_z^2/2\}/kT}\$dv_z\$\right)\}} \quad \text{\$} \\ &\text{\$end\{array\}} \end{aligned}$$

<2-42> (2.2.10)

$$\begin{aligned} &\text{\$left\$begin\{array\}\{l\}} \\ &\text{\$displaystyle\{v_x=\$left(\$frac\{2kT\}\{m\}\$right)\^{1/2}\}p\}} \quad \text{\$} \\ &\text{\$displaystyle\{v_y=\$left(\$frac\{2kT\}\{m\}\$right)\^{1/2}\}q\}} \quad \text{\$} \\ &\text{\$displaystyle\{v_z=\$left(\$frac\{2kT\}\{m\}\$right)\^{1/2}\}r\}} \\ &\text{\$end\{array\}\$right.} \end{aligned}$$

<2-43> (2.2.11)

$$\begin{aligned} &\text{\$left\$begin\{array\}\{l\}} \\ &\text{\$displaystyle\{p=\$left(\$frac\{m\}\{2kT\}\$right)\^{1/2}\}v_x\}} \quad \text{\$} \\ &\text{\$displaystyle\{q=\$left(\$frac\{m\}\{2kT\}\$right)\^{1/2}\}v_y\}} \quad \text{\$} \\ &\text{\$displaystyle\{r=\$left(\$frac\{m\}\{2kT\}\$right)\^{1/2}\}v_z\}} \\ &\text{\$end\{array\}\$right.} \end{aligned}$$

<2-44> (2.2.12)

$$\begin{aligned} &\text{\$begin\{array\}\{rl\}} \\ &\text{\$displaystyle\{\$angle K\$rangle} \\ &\&=\text{\$displaystyle\{3\$left(\$frac\{m\}\{2\$pi kT\}\$right)\^{3/2}\$times\$left(\$frac\{m\}\{2\}\$cdot\$frac\{2kT\}\{m\}\$p^2\$\right)} \\ &\&=\text{\$left[\$int_{-\infty}^{\infty}\$left(\$frac\{m\}\{2\}\$cdot\$frac\{2kT\}\{m\}\$p^2\$\right) \$e^{\{-p^2/2\}}dp\$\right]} \quad \text{\$} \\ &\&=\text{\$qqquad\$qqquad\$qqquad\$times\$displaystyle\{\$left(\$frac\{2kT\}\{m\}\$right)\^{1/2}\}\$left[\$int_{-\infty}^{\infty}\$e^{\{-p^2/2\}}dp\$\right] \$e^{\{-q^2/2\}}dq\$\right]} \\ &\&=\text{\$times\$left(\$frac\{2kT\}\{m\}\$right)\^{1/2}\}\$left[\$int_{-\infty}^{\infty}\$e^{\{-p^2/2\}}dr\$\right]\}} \quad \text{\$} \\ &\&=\text{\$displaystyle\{3\$left(\$frac\{kT\}\{\$pi\}\$right)^{3/2}\}\$right)} \\ &\&=\text{\$left[\$int_{-\infty}^{\infty}\$p^2e^{\{-p^2/2\}}dp\$\right]} \\ &\&=\text{\$times\$left[\$int_{-\infty}^{\infty}\$e^{\{-q^2/2\}}dq\$\right]} \\ &\&=\text{\$times\$left[\$int_{-\infty}^{\infty}\$e^{\{-r^2/2\}}dr\$\right]} \\ &\text{\$end\{array\}}} \end{aligned}$$

<2-45>

$$\$int_{-\infty}^{\infty}\$p^2e^{\{-p^2/2\}}dp=\$frac\{\$sqrt\{\$pi\}\}\{2\}$$

<2-46>

$$\int_{-\infty}^{\infty} e^{-q^2} dq = \int_{-\infty}^{\infty} e^{-r^2} dr = \sqrt{\pi}$$

<2-47> (2.2.13)

$$\begin{aligned} &\text{\$begin\{array\}{rl}} \\ &\text{\$langle K\$rangle} \\ &\&=\text{\$displaystyle\{3\$left(\$frac{kT}{\pi}^{3/2}\$right)\times\$left(\$frac{\sqrt{\pi}}{2}\$right)} \\ &\&\times\$left(\sqrt{\pi}\$right)^2 \quad \quad \\ &\&=\text{\$displaystyle\{3kT\}/2\}} \\ &\text{\$end\{array\}} \end{aligned}$$

<2-48>

$$\vec{v} = \vec{v}_x + \vec{v}_y + \vec{v}_z$$

<2-49> (2.2.14)

$$\begin{aligned} &\text{\$begin\{array\}{rl}} \\ &\text{\$displaystyle\{\$langle \$vec\{v\}\$rangle\}} \\ &\&=\text{\$displaystyle\{\$vec\{i\}\$left(\$frac{m}{2\pi kT}\$right)^{3/2}\}} \\ &\&\times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_x^2/2)/kT\}} dv_x\right] \quad \quad \\ &\&\&=\text{\$quad\$displaystyle\{\$times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_y^2/2)/kT\}} dv_y\right]\}} \\ &\&\&\times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_z^2/2)/kT\}} dv_z\right] \quad \quad \\ &\&+\text{\$displaystyle\{\$vec\{j\}\$left(\$frac{m}{2\pi kT}\$right)^{3/2}\}} \\ &\&\times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_x^2/2)/kT\}} dv_x\right] \quad \quad \\ &\&\&=\text{\$quad\$displaystyle\{\$times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_y^2/2)/kT\}} dv_y\right]\}} \\ &\&\&\times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_z^2/2)/kT\}} dv_z\right] \quad \quad \\ &\&+\text{\$displaystyle\{\$vec\{k\}\$left(\$frac{m}{2\pi kT}\$right)^{3/2}\}} \\ &\&\times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_x^2/2)/kT\}} dv_x\right] \quad \quad \\ &\&\&=\text{\$quad\$displaystyle\{\$times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_y^2/2)/kT\}} dv_y\right]\}} \\ &\&\&\times\$left[\int_{-\infty}^{\infty} e^{-\{(mv_z^2/2)/kT\}} dv_z\right]\} \\ &\text{\$end\{array\}} \end{aligned}$$

<2-50> (2.2.15)

$$\langle \vec{v} \rangle = 0$$

<2-51>

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

<2-52> (2.2.16)

$$\begin{aligned} &\text{\$begin\{array\}{rl}} \\ &\text{\$displaystyle\{\$langle v\$rangle\}} \\ &\&=\text{\$displaystyle\{\$left(\$frac{m}{2\pi kT}\$right)^{3/2}\int_{-\infty}^{\infty} e^{-\{(mv_x^2/2)/kT\}} dv_x\}} \\ &\&\times\$displaystyle\{\int_{-\infty}^{\infty} e^{-\{(mv_y^2/2)/kT\}} dv_y\} \quad \quad \\ &\&\&=\text{\$quad\$times\$displaystyle\{\int_{-\infty}^{\infty} e^{-\{(mv_z^2/2)/kT\}} dv_z\}} \\ &\&\&e^{-\{(mv_z^2/2)/kT\}} \\ &\text{\$end\{array\}} \end{aligned}$$

<2-53>

```
¥left¥{¥begin{array}{l}
x=r¥sin¥theta¥cos¥phi ¥¥ ¥¥
y=r¥sin¥theta¥sin¥phi ¥¥ ¥¥
z=r¥cos¥theta
¥end{array}¥right.
```

<2-54> (2.2.17)

```
¥left¥{¥begin{array}{l}
v_x=v¥sin¥theta¥cos¥phi ¥¥ ¥¥
v_y=v¥sin¥theta¥sin¥phi ¥¥ ¥¥
v_z=v¥cos¥theta
¥end{array}¥right.
```

<2-55> (2.2.18)

```
¥begin{array}{r}
¥displaystyle{\langle v \rangle}
&= \displaystyle{\left( \frac{m}{2\pi kT} \right)^{3/2} \int_0^{\infty} v^2 dv}
&\quad \int_{-\pi}^{\pi} \sin\theta d\theta \int_0^{2\pi} e^{-(mv^2/2)/kT} d\phi
&= \displaystyle{\left( \frac{m}{2\pi kT} \right)^{3/2} \int_0^{\infty} v^3}
&\quad e^{-(mv^2/2)/kT} dv \int_{-\pi}^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi
&= \displaystyle{\left( \frac{m}{2\pi kT} \right)^{3/2} 4\pi \int_0^{\infty} v^3}
&\quad e^{-(mv^2/2)/kT} dv
&= \sqrt{\frac{8kT}{\pi m}}
```

<2-56>

$K = \frac{m}{2} \langle v_x^2 + v_y^2 + v_z^2 \rangle = \frac{m}{2} \langle v^2 \rangle$

<2-57>

$\langle v^2 \rangle = \frac{3kT}{m}$

<2-58>

$\langle v^2 \rangle^2 = \frac{8kT}{m} \approx \frac{2.55kT}{m}$

<2-59>

$\sqrt{\langle v^2 \rangle - \langle v \rangle^2}$

<2-60> (2.3.1)

$P(\vec{v}; T) d\vec{v} = \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-\beta E(\vec{v})} d\vec{v}$

<2-61> (2.3.2)

$\beta = \frac{1}{kT}$

<2-62> (2.3.3)

$$\begin{aligned}\mathbb{E}\epsilon(\mathbf{v}) &= \frac{m}{2} \mathbf{v}^2 \\ &= \frac{m}{2} v_x^2 + \frac{m}{2} v_y^2 + \frac{m}{2} v_z^2\end{aligned}$$

<2-63> (2.3.4)

$$\int P(\mathbf{v}; T) d\mathbf{v} = 1$$

<2-64> (2.3.5)

$$\begin{aligned}P(\mathbf{v}; T) d\mathbf{v}_1 \dots d\mathbf{v}_N &\propto e^{-\beta E(\mathbf{v})} \\ d\mathbf{v}_1 \dots d\mathbf{v}_N &\propto e^{-\beta E(\mathbf{v})}\end{aligned}$$

<2-65> (2.3.6)

$$E(\mathbf{v}) = \frac{m}{2} (v_1^2 + v_2^2 + \dots + v_N^2)$$

<2-66> (2.3.7)

$$\begin{aligned}P(\mathbf{v}; T) d\mathbf{v}_1 \dots d\mathbf{v}_N &= C \left[ e^{-\beta E(\mathbf{v})} \right] \\ e^{-\beta E(\mathbf{v})} d\mathbf{v}_1 \dots d\mathbf{v}_N &\propto e^{-\beta E(\mathbf{v})}\end{aligned}$$

<2-67> (2.3.8)

$$\int P(\mathbf{v}) d\mathbf{v}_1 \dots d\mathbf{v}_N = 1$$

<2-68> (2.3.9)

$$\begin{aligned}P[\mathbf{r}, \mathbf{v}; T] d\Gamma &= C \left[ e^{-\beta E(\mathbf{r}, \mathbf{v})} \right] d\Gamma \\ e^{-\beta E(\mathbf{r}, \mathbf{v})} d\Gamma &\propto e^{-\beta E(\mathbf{r}, \mathbf{v})}\end{aligned}$$

<2-69> (2.3.10)

$$\begin{aligned}d\mathbf{v}_1 \dots d\mathbf{v}_N &\equiv d\Gamma \\ d\mathbf{v}_1 \dots d\mathbf{v}_N &\propto d\Gamma\end{aligned}$$

<2-70> (2.3.11)

$$\int \Gamma d\Gamma = 1$$

<2-71> (2.3.12)

$$W[E(\mathbf{r}, \mathbf{v})] e^{-\beta E(\mathbf{r}, \mathbf{v})}$$

<2-72> (2.3.13)

$$x = e^{\ln x}$$

<2-73> (2.3.14)

$$\begin{aligned}W[e^{-\beta E}] &= e^{\ln W} e^{-\beta E} \\ e^{-\beta E} &\propto e^{-\beta E - kT \ln W} \\ \end{aligned}$$

<2-74> (2.3.15)

$$\begin{aligned} \text{\$begin\{array\}{rl}} \\ C &= \text{\$displaystyle\{\frac{1}{\text{\$displaystyle\{\int_V \exp[-\beta E(\vec{r}, \vec{v}) - kT \ln W(\vec{r}, \vec{v})] d\Gamma}\}}\}} \\ &\quad \text{\$end\{array\}} \\ \&= \text{\$displaystyle\{\frac{1}{Z(T)}\}} \end{aligned}$$

<2-75> (2.3.16)

$$\begin{aligned} Z(T) &\equiv \text{\$int_V e^{-\beta [E(\vec{r}, \vec{v}) - kT \ln W(\vec{r}, \vec{v})]} d\Gamma} \\ &\equiv \text{\$int_V e^{-\beta E(\vec{r}, \vec{v})} P[\vec{r}, \vec{v}; T] d\Gamma} \end{aligned}$$

<2-76> (2.3.17)

$$\text{\$int_V P[\vec{r}, \vec{v}; T] d\Gamma} \equiv U$$

<2-77> (2.3.18)

$$k \text{\$int_V} \ln W(\vec{r}, \vec{v}) P[\vec{r}, \vec{v}; T] d\Gamma \equiv S$$

<2-78>

$$E - kT \ln W$$

<2-79>

$$U - ST \equiv F(T, V)$$

<2-80> (2.3.20)

$$\frac{de^{ax}}{dx} = ae^{ax}$$

<2-81> (2.3.21)

$$\begin{aligned} \frac{dZ(T)}{d\beta} &= - \text{\$int_V} E(\vec{r}, \vec{v}) \\ &\quad e^{-\beta E(\vec{r}, \vec{v})} d\Gamma \end{aligned}$$

<2-82> (2.3.22)

$$\begin{aligned} \text{\$begin\{array\}{rl}} \\ \frac{1}{Z(T)} \frac{dZ(T)}{d\beta} &= - \text{\$int_V} E(\vec{r}, \vec{v}) \\ &\quad e^{-\beta E(\vec{r}, \vec{v})} d\Gamma \\ &\quad \text{\$frac{e^{-\beta E(\vec{r}, \vec{v})}}{Z(T)} d\Gamma} \\ &\quad \text{\$end\{array\}} \\ \&= - \text{\$int_V} E(\vec{r}, \vec{v}) P[\vec{r}, \vec{v}; T] d\Gamma \end{aligned}$$

<2-83> (2.3.23)

$$\begin{aligned} \text{\$begin\{array\}{rl}} \\ U &\&= - \text{\$displaystyle\{\frac{1}{Z(T)} \frac{dZ(T)}{d\beta}\}} \\ &\&= - \text{\$displaystyle\{\frac{d \ln Z(T)}{d\beta}\}} \end{aligned}$$

$\$end{array}$

<2-84> (2.3.24)

$$\frac{d \ln y(x)}{dx} = \frac{1}{y} \frac{dy}{dx}$$

<2-85>

$$f(-x) = f(x)$$

<2-86>

$$I = \int_{-\infty}^{\infty} f(x) dx$$

<2-87>

$$I = \int_{-\infty}^{0} f(x) dx + \int_{0}^{\infty} f(x) dx$$

<2-88>

$$dx = -dx^{\prime}$$

<2-89>

$$\begin{aligned} \int_{-\infty}^{0} f(x) dx &= \int_{-\infty}^{0} f(-x^{\prime}) (-dx^{\prime}) \\ &= - \int_{-\infty}^{0} f(x^{\prime}) dx^{\prime} \\ &\quad + \int_{0}^{\infty} f(x) dx \end{aligned}$$

<2-90>

$$\int_a^b F(x) dx = - \int_b^a F(x) dx$$

<2-91>

$$\int_{-\infty}^{0} f(x) dx = \int_0^{\infty} f(x^{\prime}) dx^{\prime} = \int_0^{\infty} f(x) dx$$

<2-92>

$$\int_{-\infty}^{0} f(x) dx = 2 \int_0^{\infty} f(x) dx$$

<2-93>

$$\begin{aligned} &\$begin{array}{l} \\ \$displaystyle \left( \frac{m}{2kT} \right)^{3/2} \int_{-\infty}^{\infty} e^{-\frac{(mv_x^2/2)/kT}{}} dv_x \right) \\ &\quad \times \int_{-\infty}^{\infty} e^{-\frac{(mv_y^2/2)/kT}{}} dv_y \right) \cdot \cdot \cdot \\ &\quad \&quad \cdot \cdot \cdot \int_{-\infty}^{\infty} e^{-\frac{(mv_z^2/2)/kT}{}} dv_z \right) = 1 \\ &\$end{array} \end{aligned}$$

<2-94>

$$\begin{aligned} &\left( \frac{m}{2kT} \right)^{3/2} \\ &\left( \int_{-\infty}^{\infty} e^{-\frac{(mv_x^2/2)/kT}{}} dv_x \right)^3 = 1 \end{aligned}$$

<2-95>

$$q = \left( \frac{m}{2kT} \right)^{1/2} v_x$$

<2-96>

$$dv_x = \left( \frac{2kT}{m} \right)^{1/2} dq$$

<2-97>

$$\begin{aligned} \text{the left side} &= \left( \frac{m}{2\pi kT} \right)^{3/2} \\ &\times \left[ \left( \frac{2kT}{m} \right)^{1/2} \int_{-\infty}^{\infty} e^{-q^2/2} dq \right]^3 \end{aligned}$$

<2-98>

$$\begin{aligned} \text{the left side} &= \left( \frac{m}{2\pi} \right)^{3/2} \\ &\times \left[ \left( \frac{2kT}{m} \right)^{1/2} \sqrt{\pi} \right]^3 = 1 \end{aligned}$$

<2-99>

$$\left( \frac{mN^{2/3}}{2\pi kT} \right)^{3/2}$$